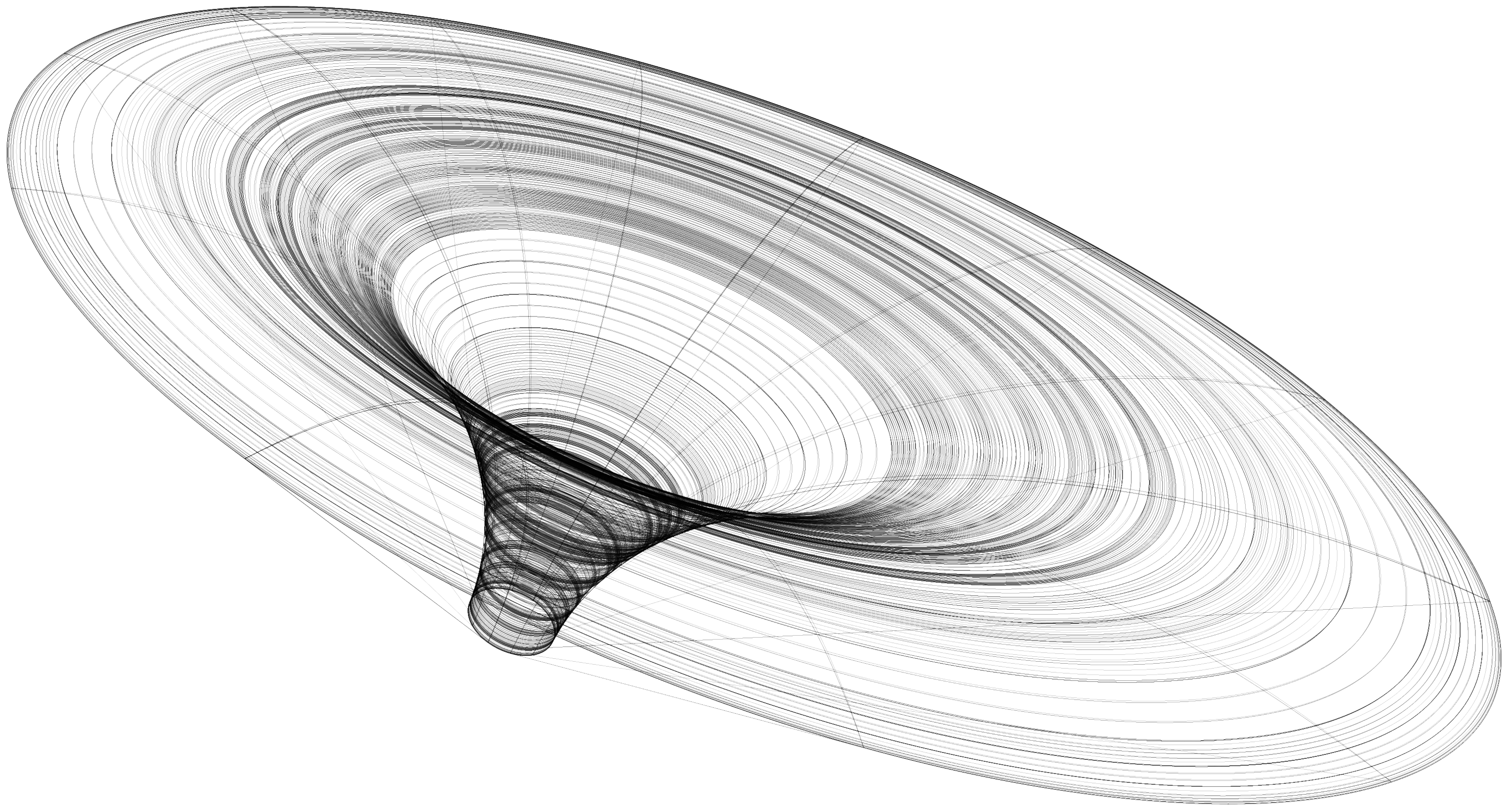

2022.06.07 @ 立教大学理論物理学研究室セミナー

原始ブラックホールのピーク理論と非ガウス尾

多田 祐一郎　名古屋大学・KEK

w/ K.T. Abe, A. Escrivà, R. Inui, N. Kitajima, S. Yokoyama, C. M. Yoo

2109.00791, 2202.01028, in prep.

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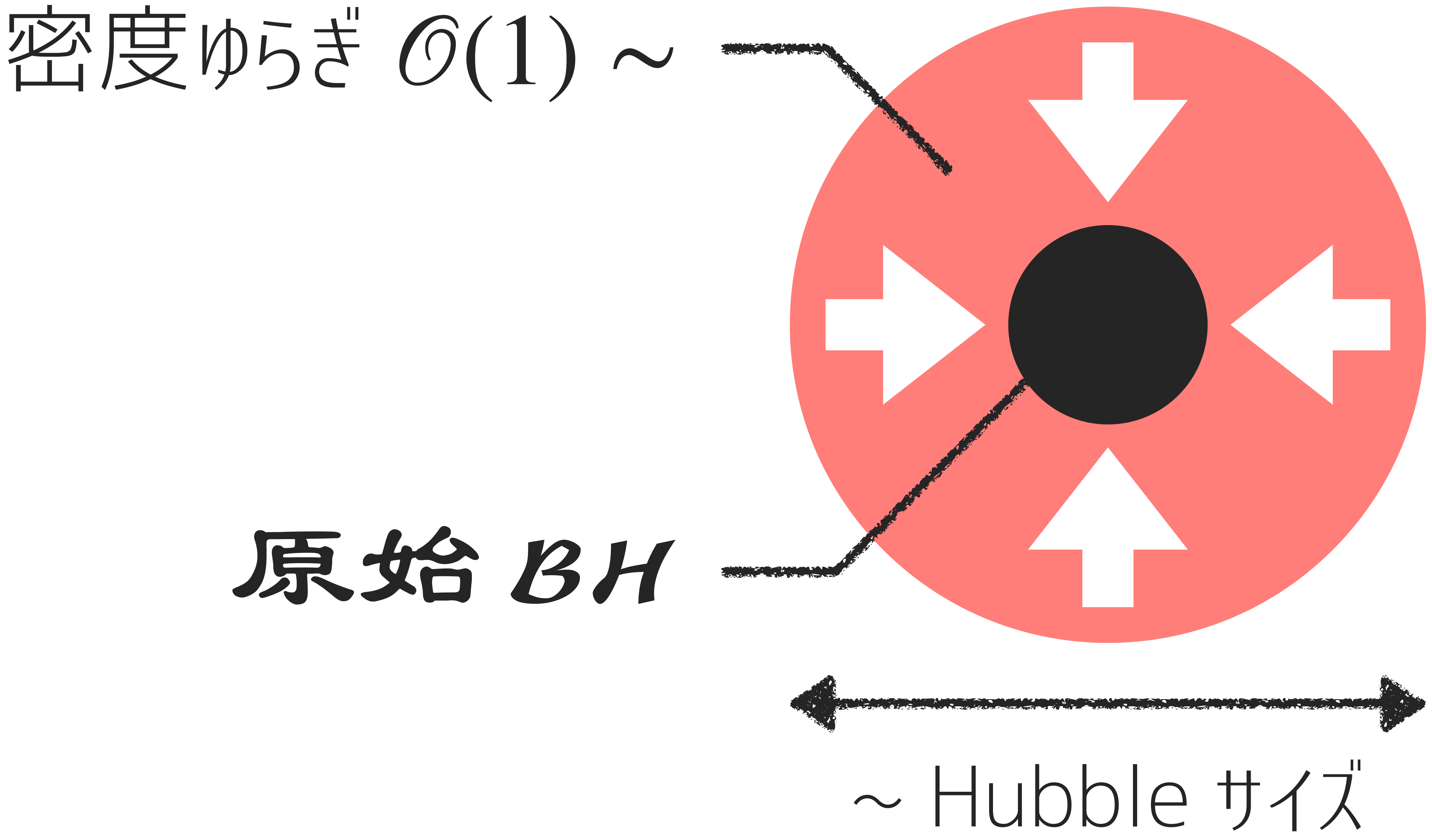
原始ブラックホールのピーク理論と非ガウス尾

1. 原始ブラックホール
2. 曲率ゆらぎの非ガウス尾
3. ピーク理論
4. 誘導背景重力波

原始 BH

Carr & Hawking ’71, ’74, ‘75

放射優勢宇宙



- 暗黒物質?
- LVK 合体重力波?
- 超大質量 BH の種?
- OGLE 重力レンズ天体?
- 第 9 惑星?
- r過程の引き金?

⋮

存在量？

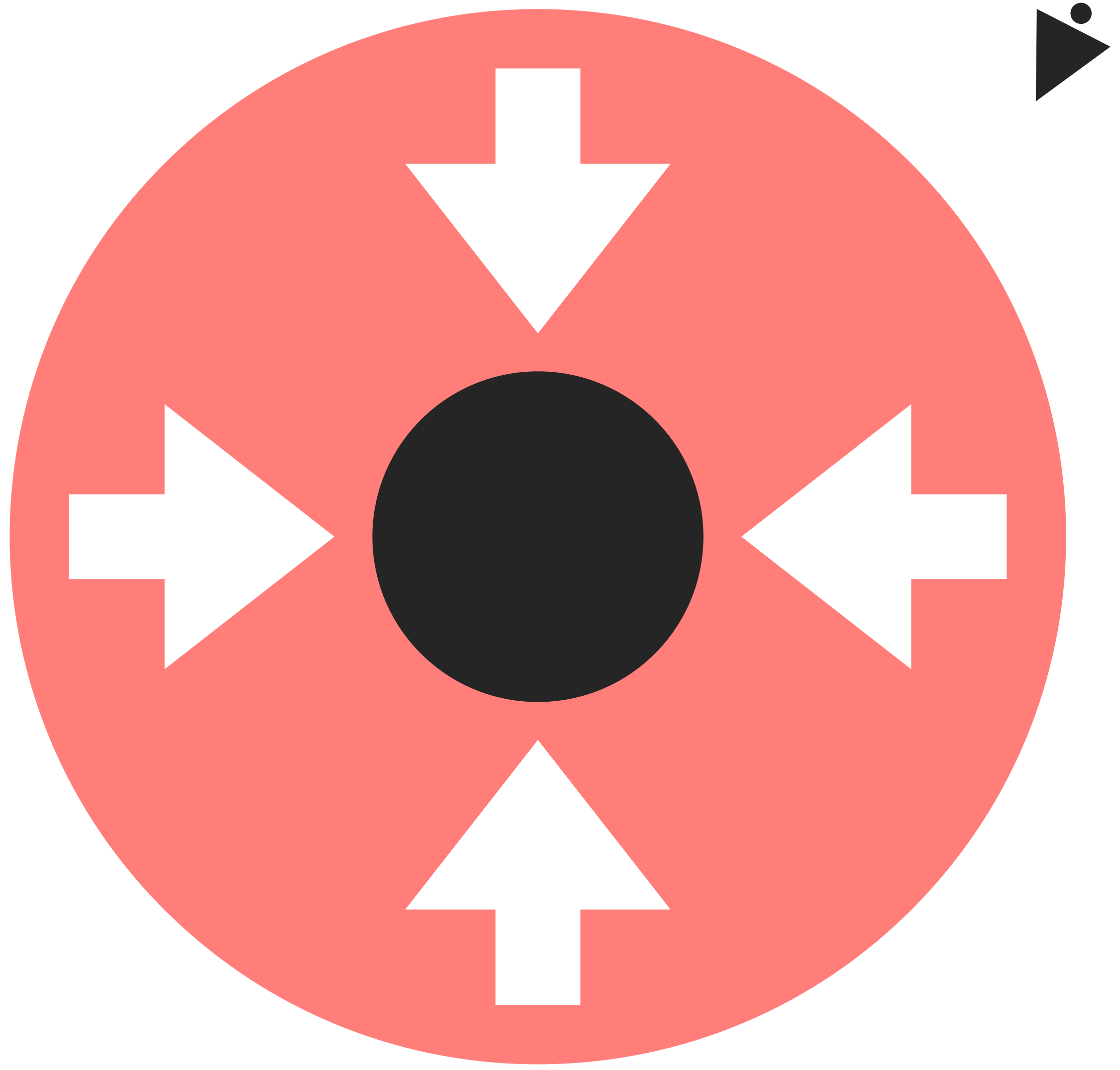
インフレーション

YT & Vennin '21

大きい初期ゆらぎ

Escrivà, Kitajima, YT,
Yokoyama, Yoo
'21, '22

原始 BH

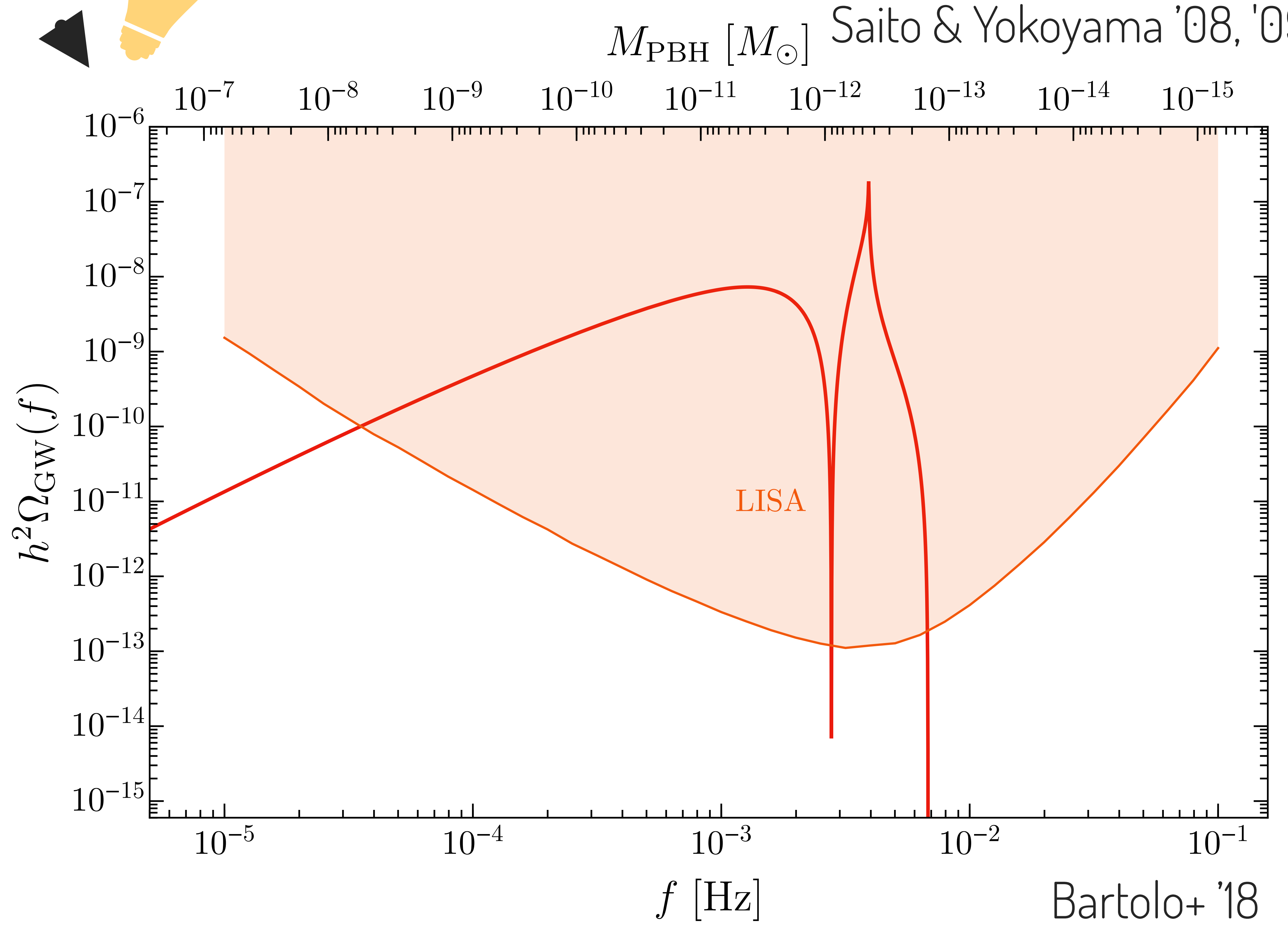


クロスチェック

Abe, Inui, YT, Yokoyama
in prep.

2次誘導背景重力波

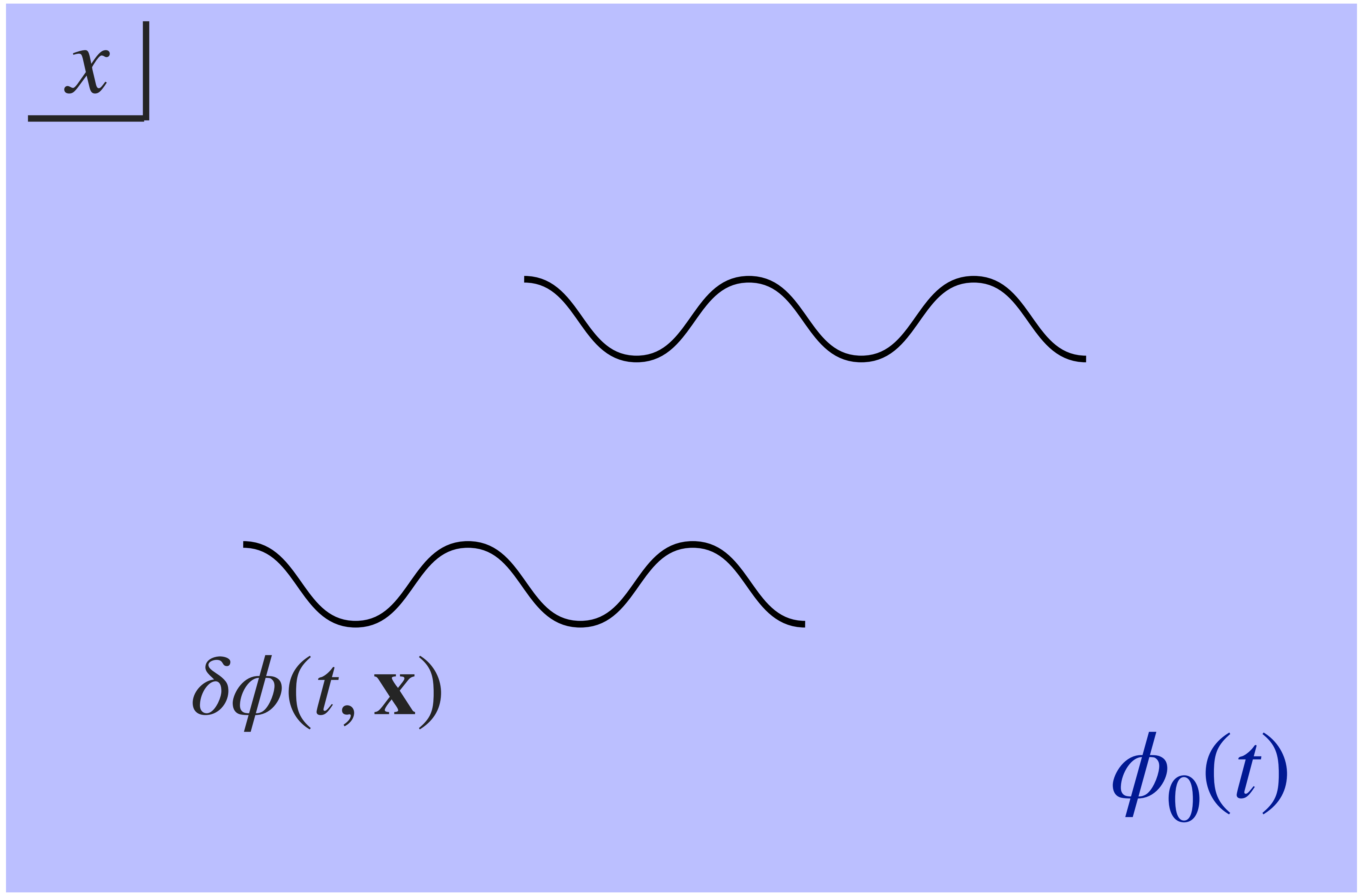
Saito & Yokoyama '08, '09



Bartolo+ '18

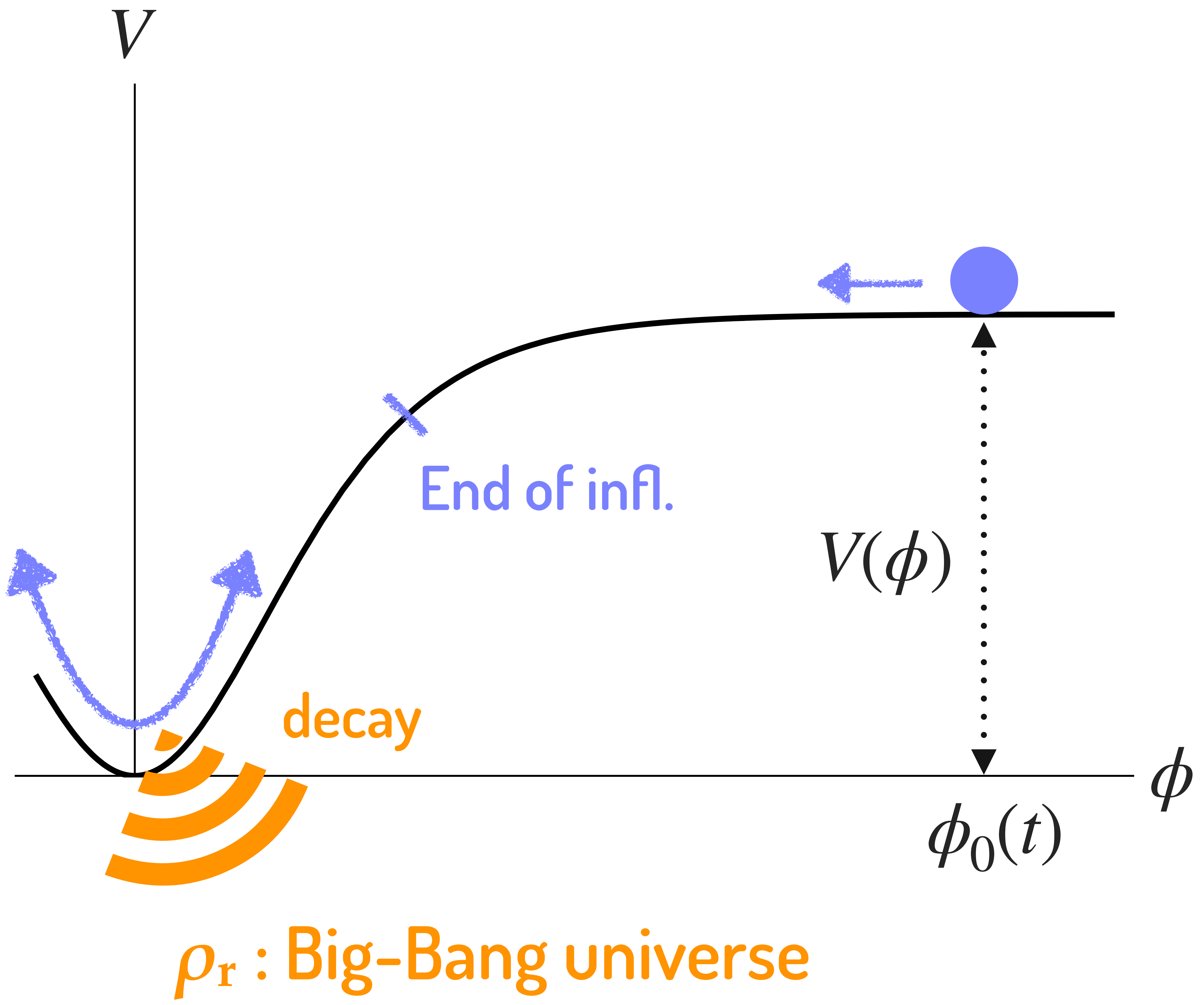
Heavy-tailed Curvature Perturbation

δN 形式

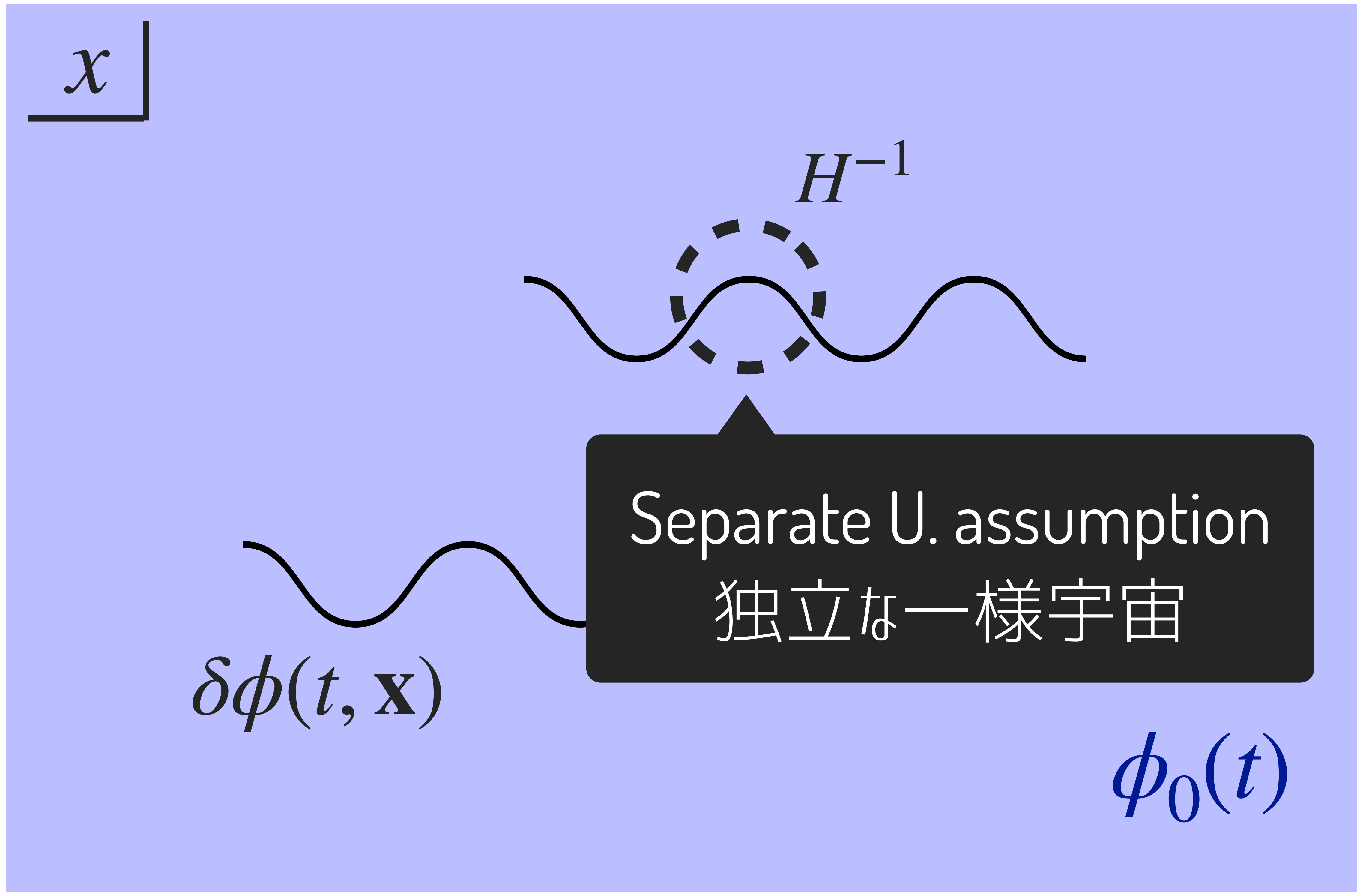


- 線形摂動理論

$$
\begin{cases}
\phi(\tau, \mathbf{x}) = \phi_0(\tau) + \delta\phi(\tau, \mathbf{x}) \\
g_{\mu\nu}(\tau, \mathbf{x}) = a^2(\tau)\eta_{\mu\nu} + h_{\mu\nu}(\tau, \mathbf{x})
\end{cases}
$$

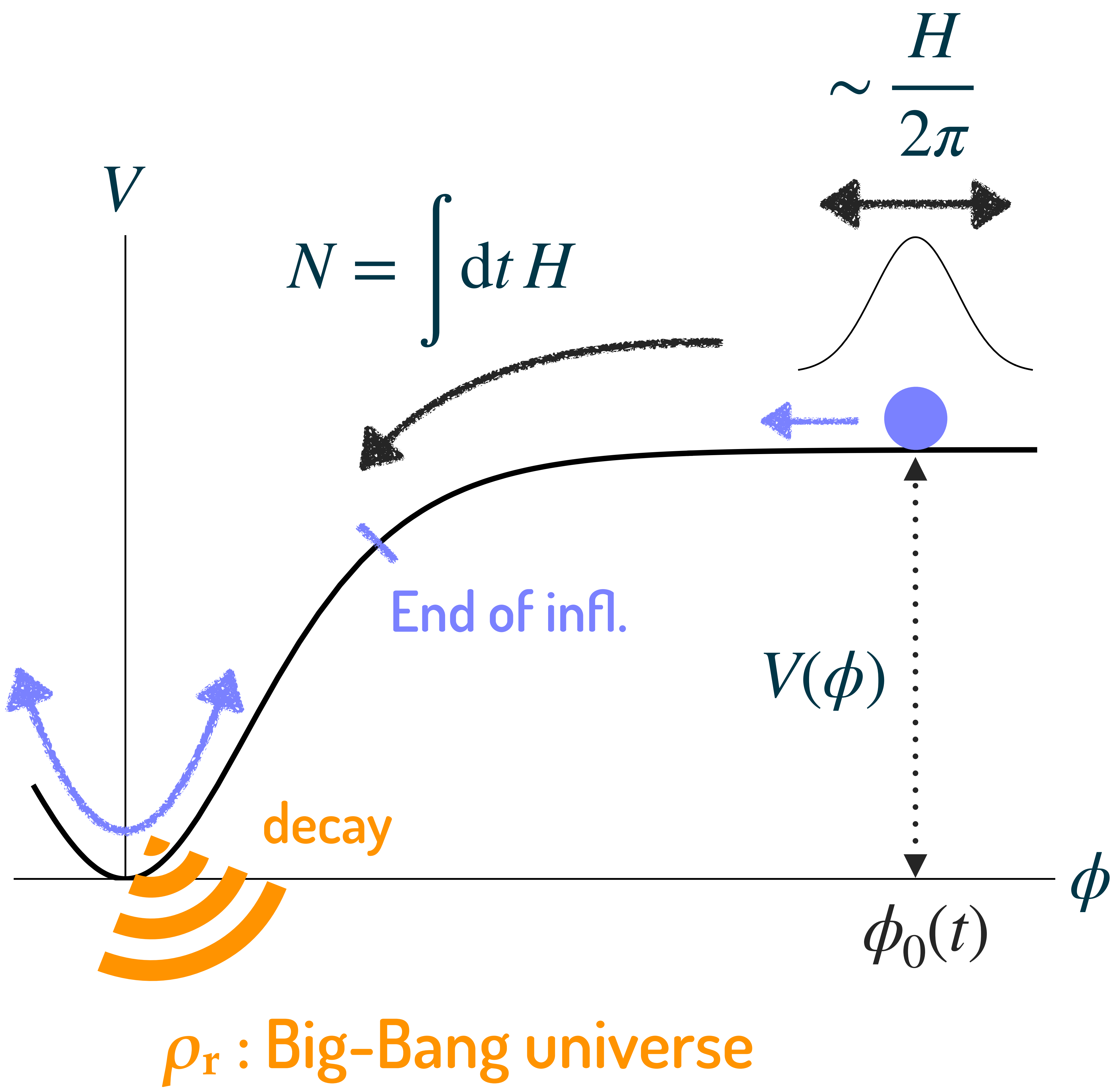


δN 形式



- δN 形式 (非摂動) Lyth, Malik, Sasaki '04

$$\zeta = \delta N = N(\phi_0 + \delta\phi) - N(\phi_0)$$
$$= -H\frac{\delta\rho}{\dot{\rho}_0} \quad \text{(flat slice)}$$



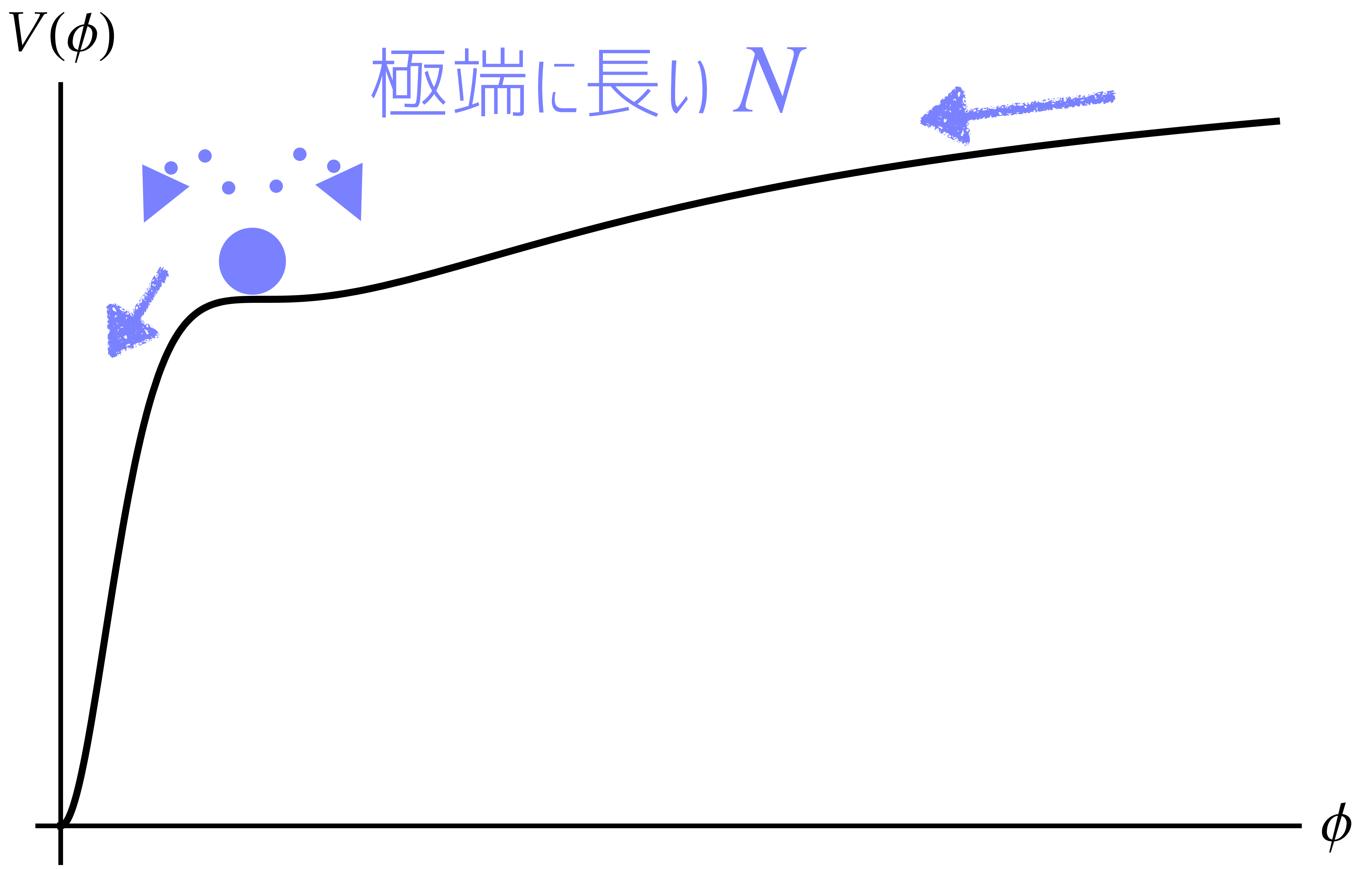
ガウシアン？

- for small $\zeta \ll 1$

  $\zeta \simeq N_\phi \delta\phi$: Gaussian ✔

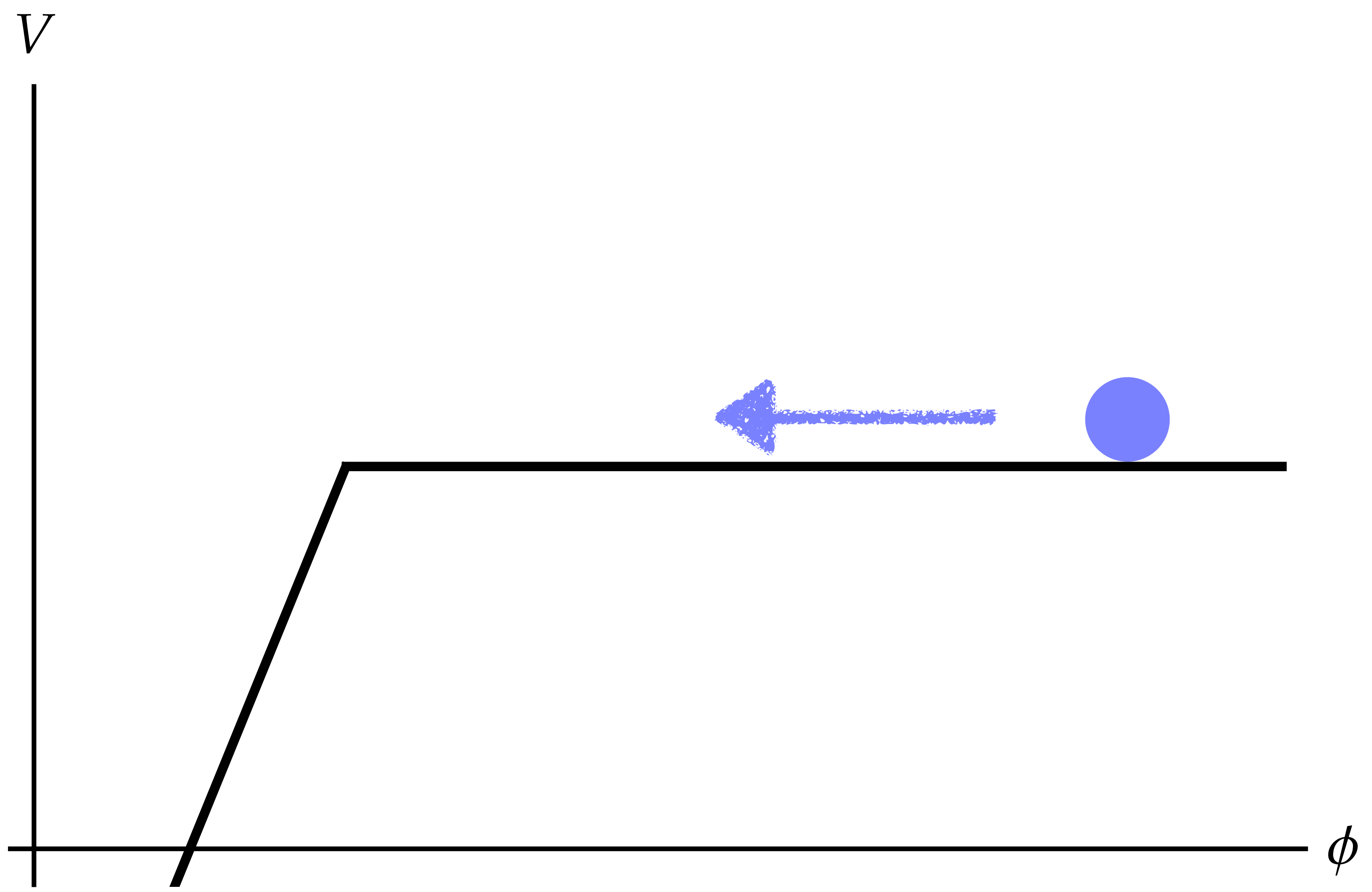
- for large $\zeta \sim 1$

  $\zeta = \zeta_{\mathrm{NL}}(\delta\phi)$: Gaussian ?



Exp.-tail

- Exact Ultra SR



$$\ddot{\phi} + 3H\dot{\phi} = 0 \;\Rightarrow\; \dot{\phi} = \dot{\phi}_0 e^{-3Ht}$$

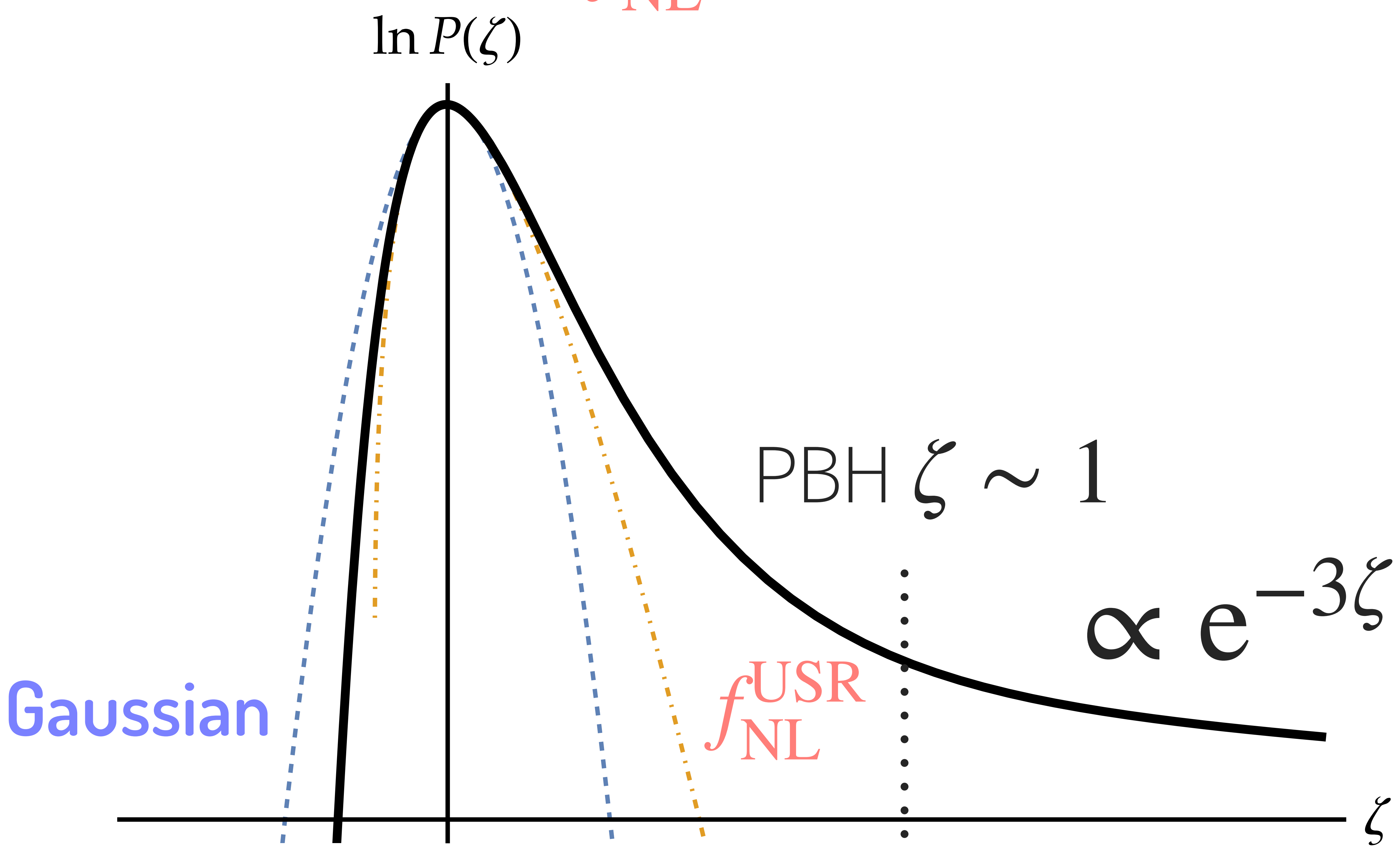
$$\zeta = \delta N = -\frac{1}{3}\ln\left(1 - 3H\frac{\delta\phi}{|\dot{\phi}_{\mathrm{f}}|}\right)$$

Cai+ '18

$$\zeta = -\frac{1}{3}\ln\left(1 - 3\zeta_{\mathrm{G}}\right) \Leftrightarrow \zeta_{\mathrm{G}} = \frac{1}{3}\left(1 - e^{-3\zeta}\right)$$

$$= \zeta_{\mathrm{G}} + \frac{3}{5} \times \underbrace{\boxed{\frac{5}{2}}}_{f_{\mathrm{NL}}^{\mathrm{USR}}}\zeta_{\mathrm{G}}^2 + \cdots$$

Atal+ '19
Ezquiaga+ '19
Biagetti+ '21

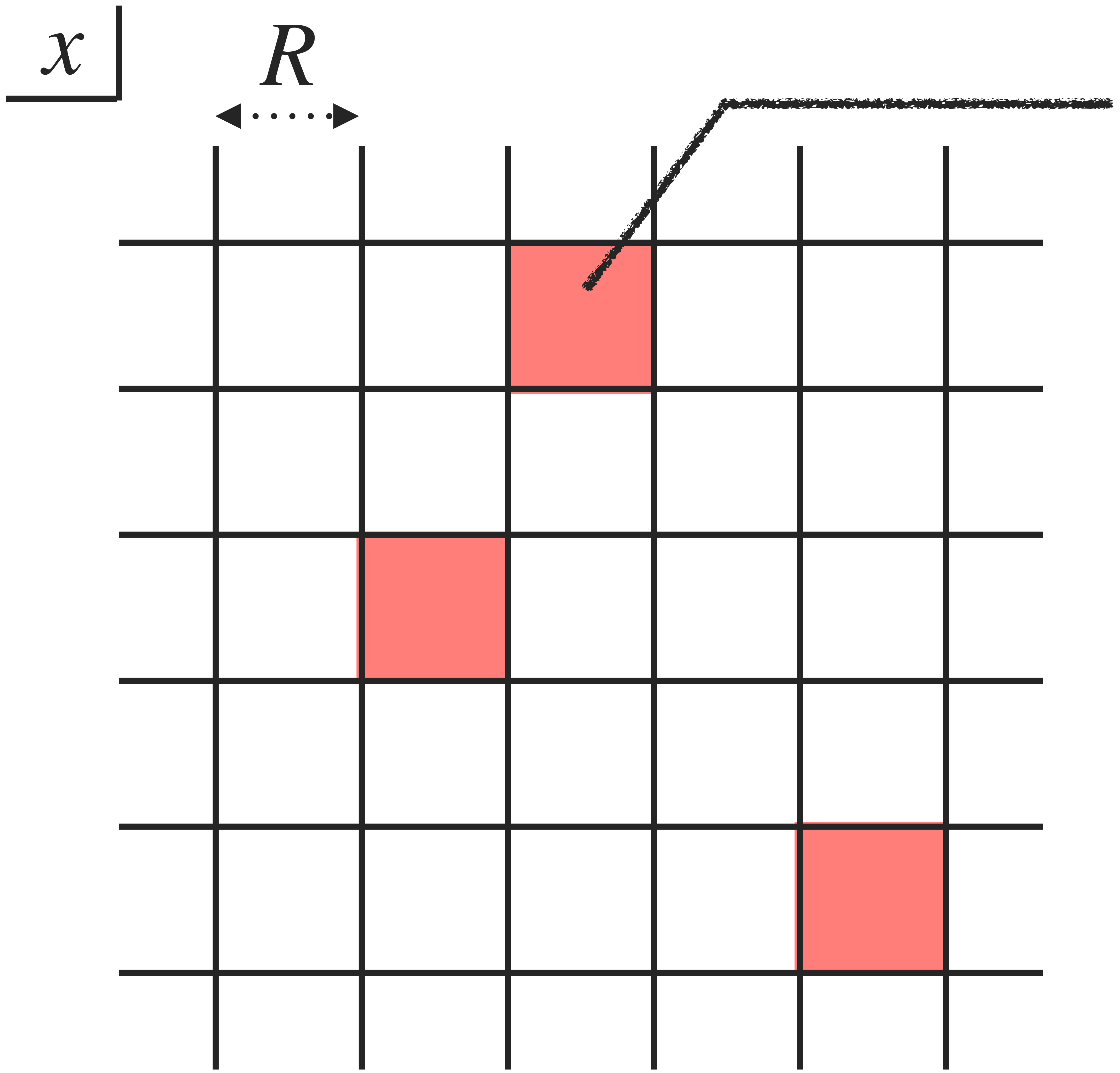


Peak theory for PBH

質量関数

最も単純なアプローチ

Carr '75 (Press & Schechter'74)



$$\delta_R(\mathbf{x}) = \int \mathrm{d}^3y\, W_R(\mathbf{x}-\mathbf{y})\delta(\mathbf{y}) \gtrsim \frac{1}{3}\left(= \frac{p}{\rho}\right)$$

➡ 原始 BH !!

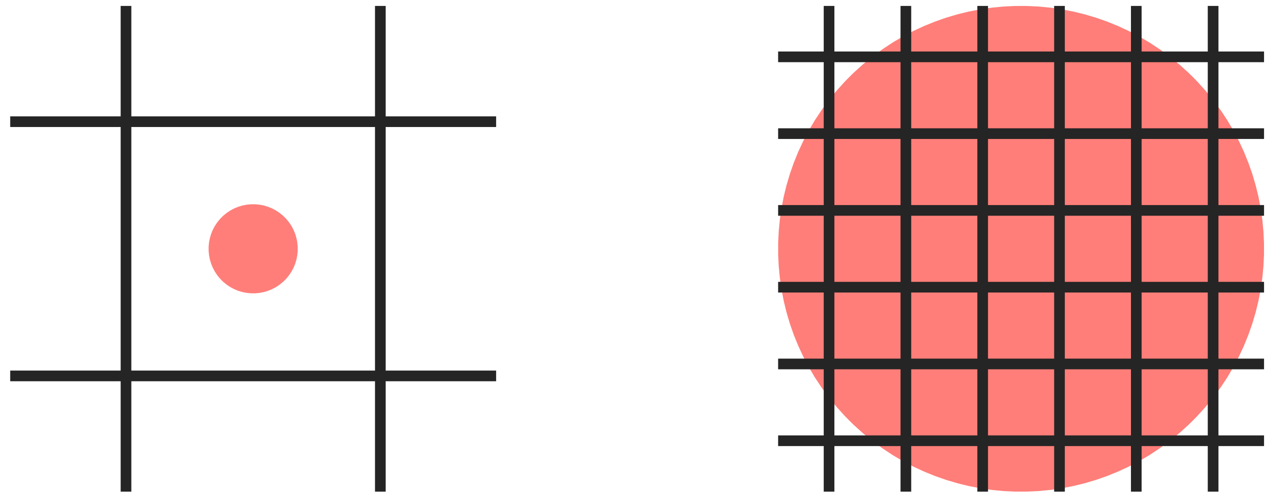
* 存在量 : $\dfrac{\rho_{\mathrm{PBH}}}{\rho_{\mathrm{tot}}} = \displaystyle\int_{1/3}^{\infty} \frac{1}{\sqrt{2\pi\sigma_R^2}} \mathrm{e}^{-\delta_R^2/2\sigma_R^2}$
* 質量 : $M_{\mathrm{PBH}} \sim M_H\Big|_{R=H^{-1}} = \dfrac{4\pi}{3}\rho R^3\Big|_{R=H^{-1}}$

質量関数

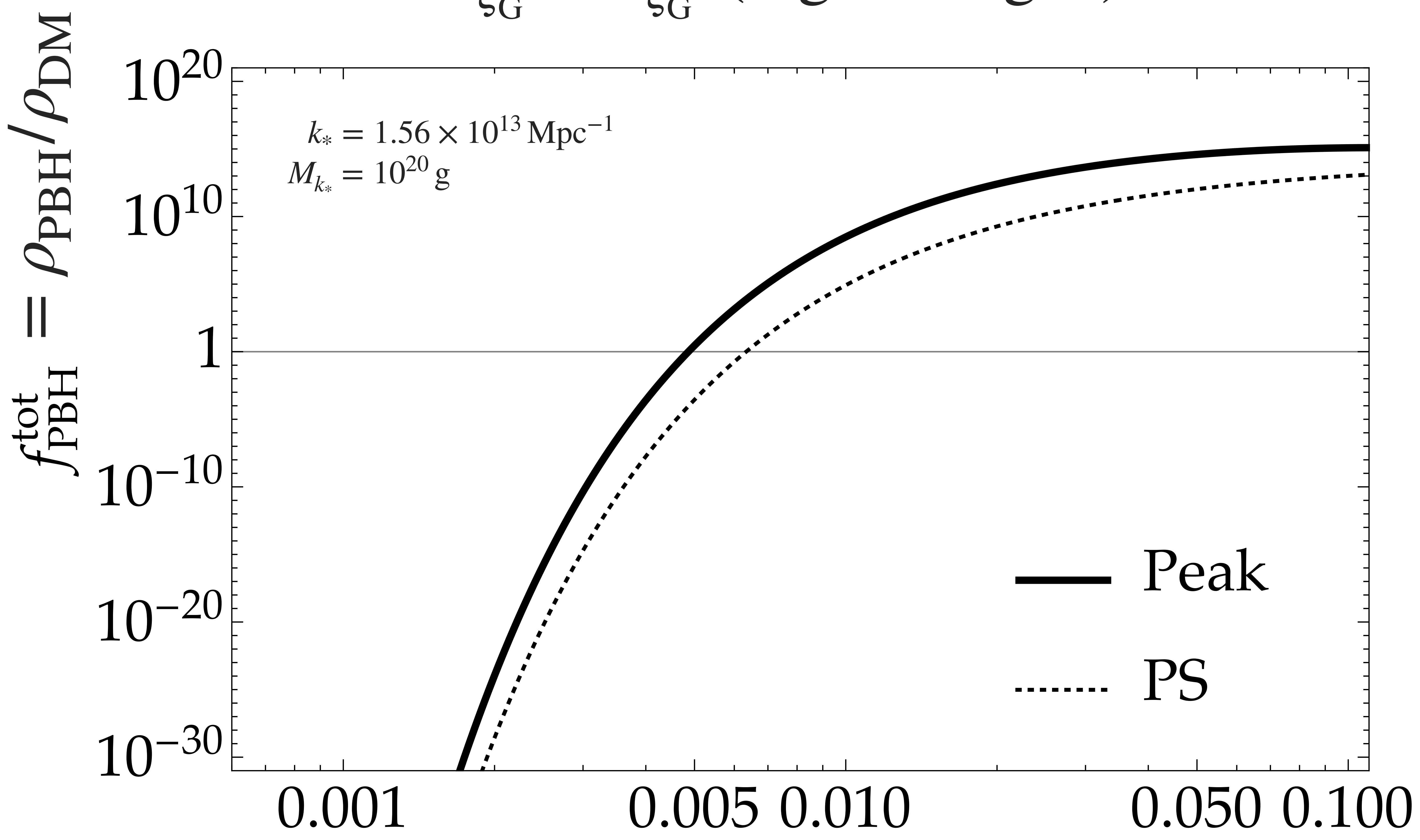
最も単純なアプローチ

Carr '75 (Press & Schechter'74)

- いつでも "$1/3$"?
- 窓関数は何を使う？
- δ_R はガウス乱数？
- $M_{\mathrm{PBH}} \overset{?}{\sim} M_H\Big|_{R=H^{-1}}$
- ピークのカウントは正しい？



$$\mathscr{P}_{\zeta_{\mathrm{G}}} = A_{\zeta_{\mathrm{G}}}\delta(\log k - \log k_*)$$



$A_{\zeta_{\mathrm{G}}}$ Kitajima, YT, Yokoyama, Yoo '21

$$\left(\text{cf. } \Omega_{\mathrm{GW}} \sim A_\zeta^2 \Omega_{\mathrm{r}}\right)$$

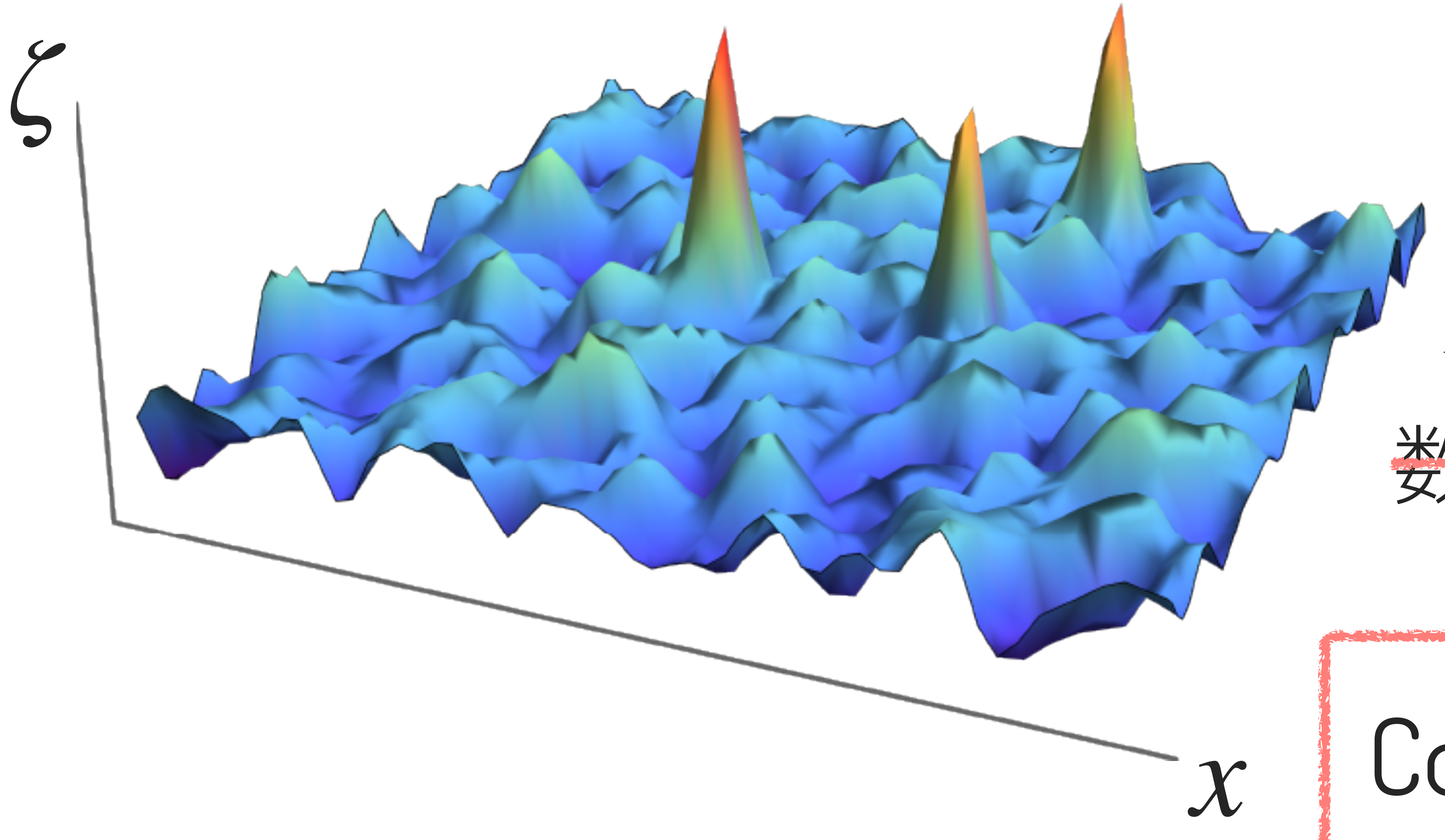
第一原理

インフレーション理論

初期ゆらぎの厳密な統計性

ピーク理論

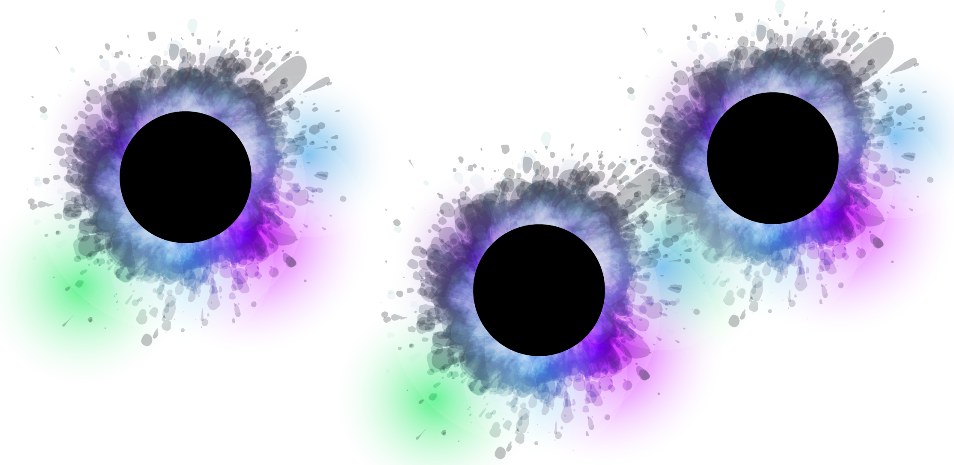
~~シミュレート~~



~~数値 GR~~

Compaction Func.

x

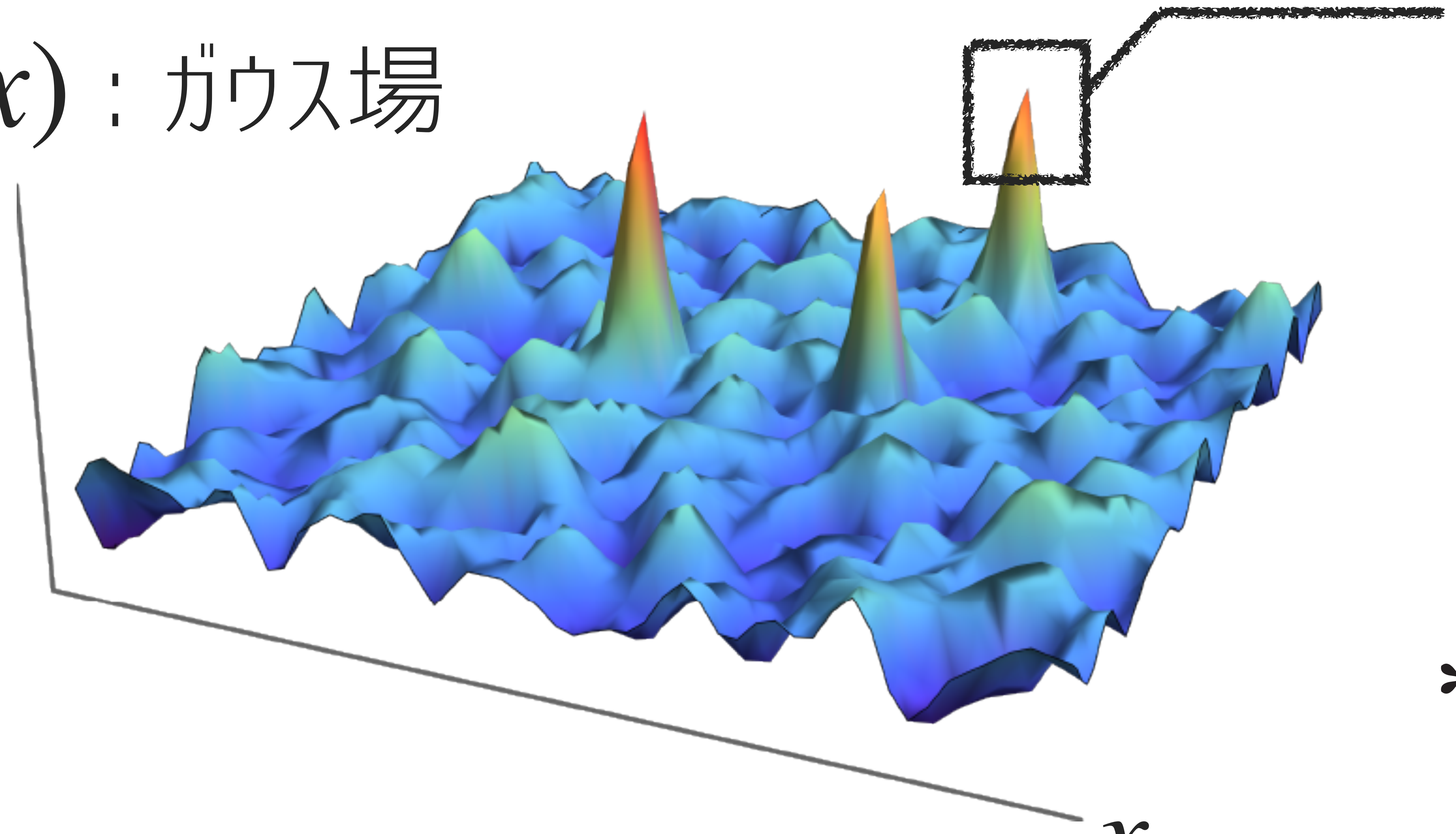


原始 $\mathcal{BH}$

ピーク理論

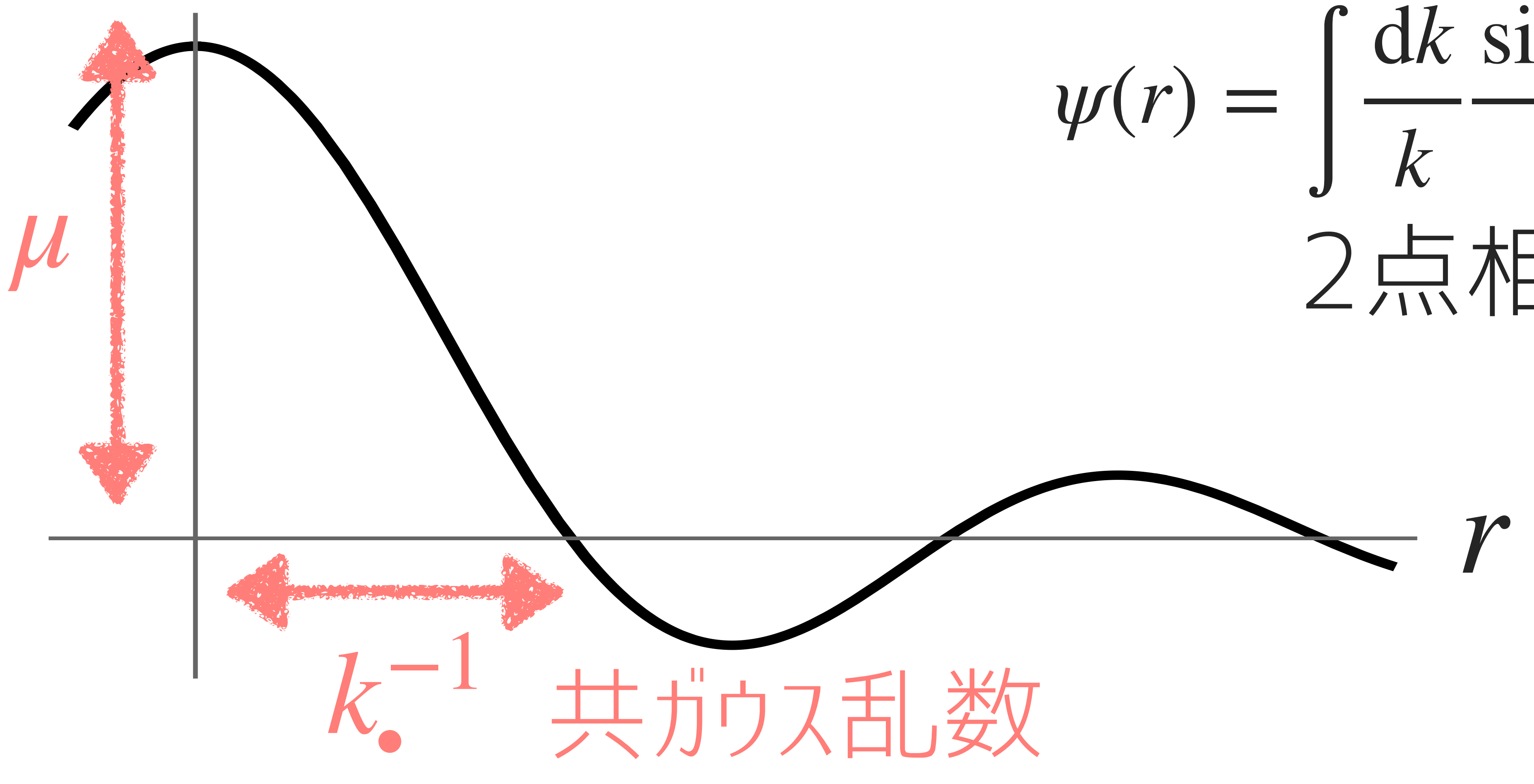
Bardeen, Bond, Kaiser, Szalay '86
Yoo, Harada, Garriga, Kohri '18
Yoo, Gong, Yokoyama '19
Yoo, Harada, Hirano, Kohri '20

$g(x)$: ガウス場



＊ 典型的な球対称ピーク形

$$\hat{g}(r) = \hat{g}\left(\psi(r); \mu, k_\bullet\right)$$



$$\psi(r) = \int \frac{\mathrm{d}k}{k} \frac{\sin kr}{kr} \mathcal{P}_g(k)$$

2点相関関数

＊ 実空間数密度

$$n_{\mathrm{pk}}(\mu, k_\bullet)\, \mathrm{d}\mu \mathrm{d}k_\bullet$$

Compaction Func.

Shibata & Sasaki '99
Harada, Yoo, Nakama, Koga '15

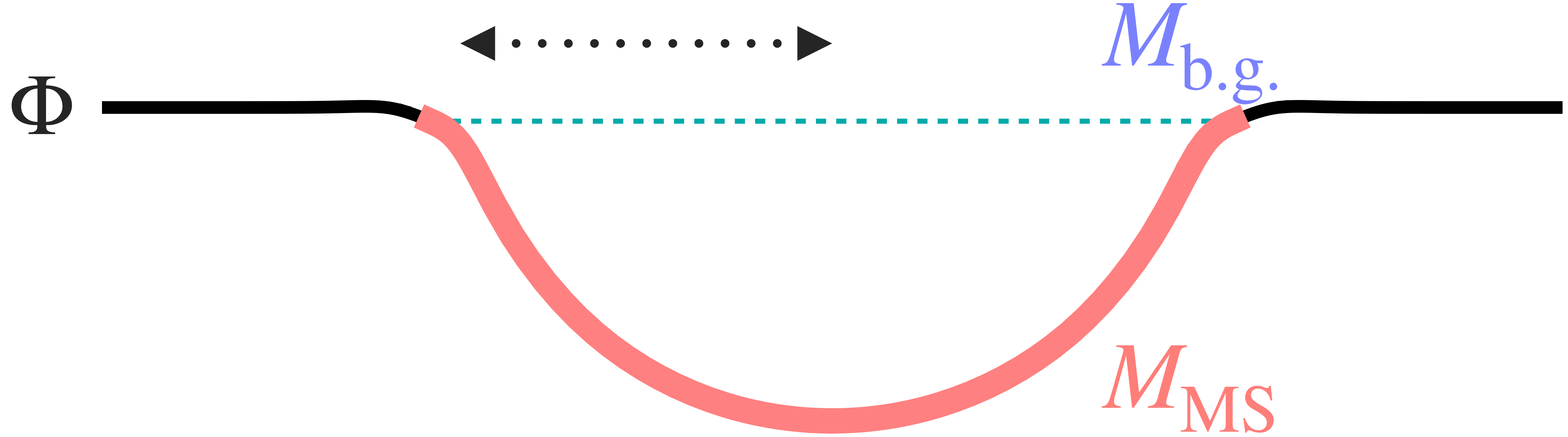
$$\delta = -\frac{8}{9}\frac{1}{a^2H^2}\mathrm{e}^{-5\zeta/2}\Delta\mathrm{e}^{\zeta/2}$$

$$\mathscr{C} = G\frac{M_{\mathrm{MS}} - M_{\mathrm{b.g.}}}{R} = \frac{1}{V(R)}\int_0^R \delta\times 4\pi R^2 \mathrm{d}R\,\bigg|_{R=H^{-1}} = \frac{2}{3}\left[1-(1+r\zeta')^2\right] \overset{?}{>} \mathscr{C}_{\mathrm{th}}$$

密度ゆらぎの平均

超 Hubble 保存

$$R = a\mathrm{e}^{\zeta}r$$



Compaction Func.

Shibata & Sasaki '99
Harada, Yoo, Nakama, Koga '15

$$\delta = -\frac{8}{9}\frac{1}{a^2H^2}\mathrm{e}^{-5\zeta/2}\Delta\mathrm{e}^{\zeta/2}$$

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密度ゆらぎの平均

超 Hubble 保存

Compaction Func. の平均はほぼ普遍的指標

$$\bar{\mathscr{C}} = \frac{1}{V(R)}\int_0^R \mathscr{C} \times 4\pi R^2 > \bar{\mathscr{C}}_{\mathrm{th}} \simeq \frac{2}{5}$$

$$\left(\to \mu > \mu_{\mathrm{th}}(k_\bullet, \cdots)\right)$$

- $f_{\mathrm{NL}} > 0$, exp.-tail, ...

  Atal, Cid, Escrivà, Garriga '19
  Escrivà, Germani, Sheth '19

- fitting for $f_{\mathrm{NL}} < 0$

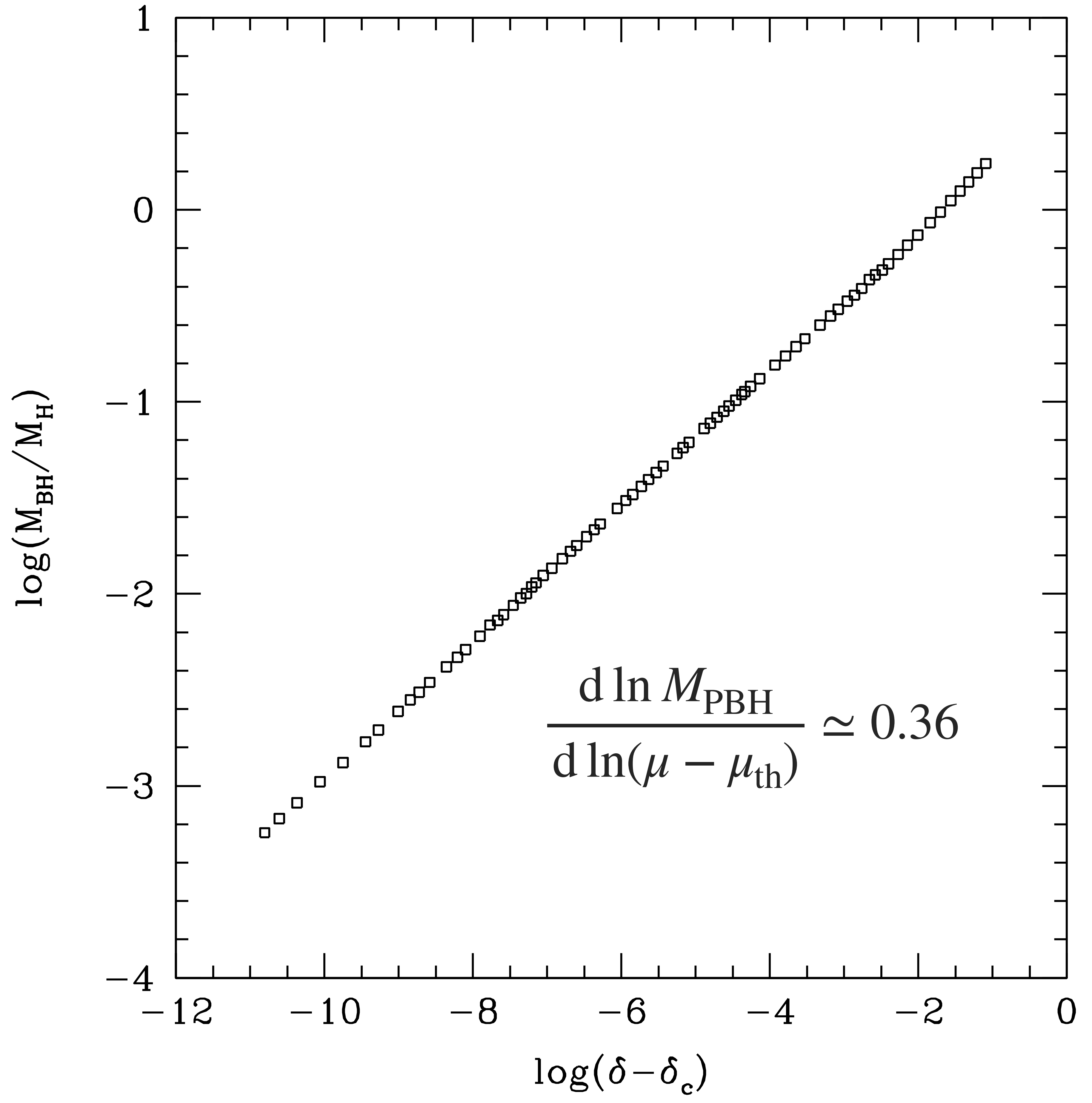
  Escrivà, YT, Yokoyama, Yoo, '22

質量

Choptuik '93
Niemeyer & Jedamzik '94, '97

* スケール則

$$M_{\mathrm{PBH}} \simeq \left(\mu - \mu_{\mathrm{th}}(k_\bullet, \cdots)\right)^{0.36} M_H \Big|_{R=H^{-1}}$$

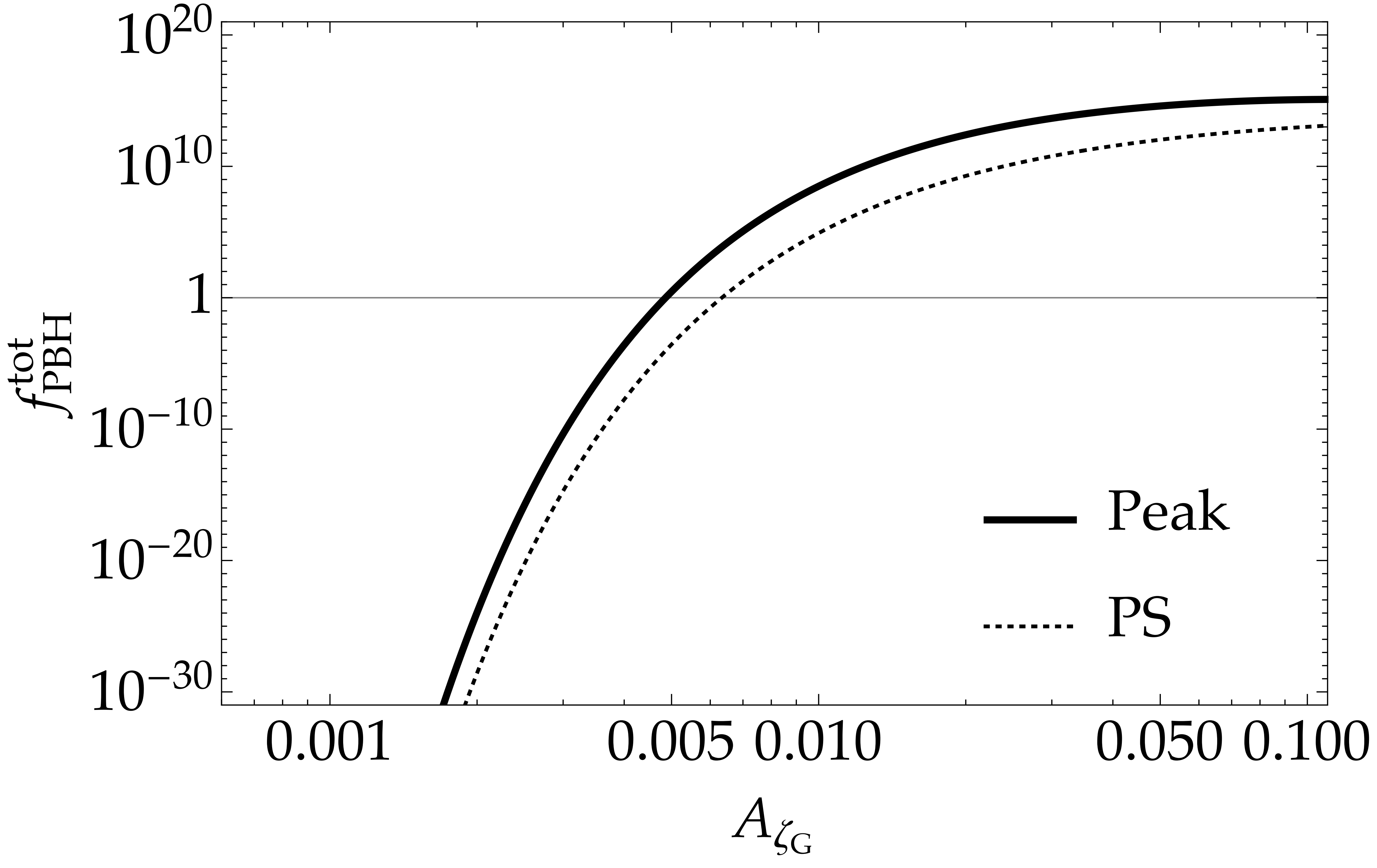


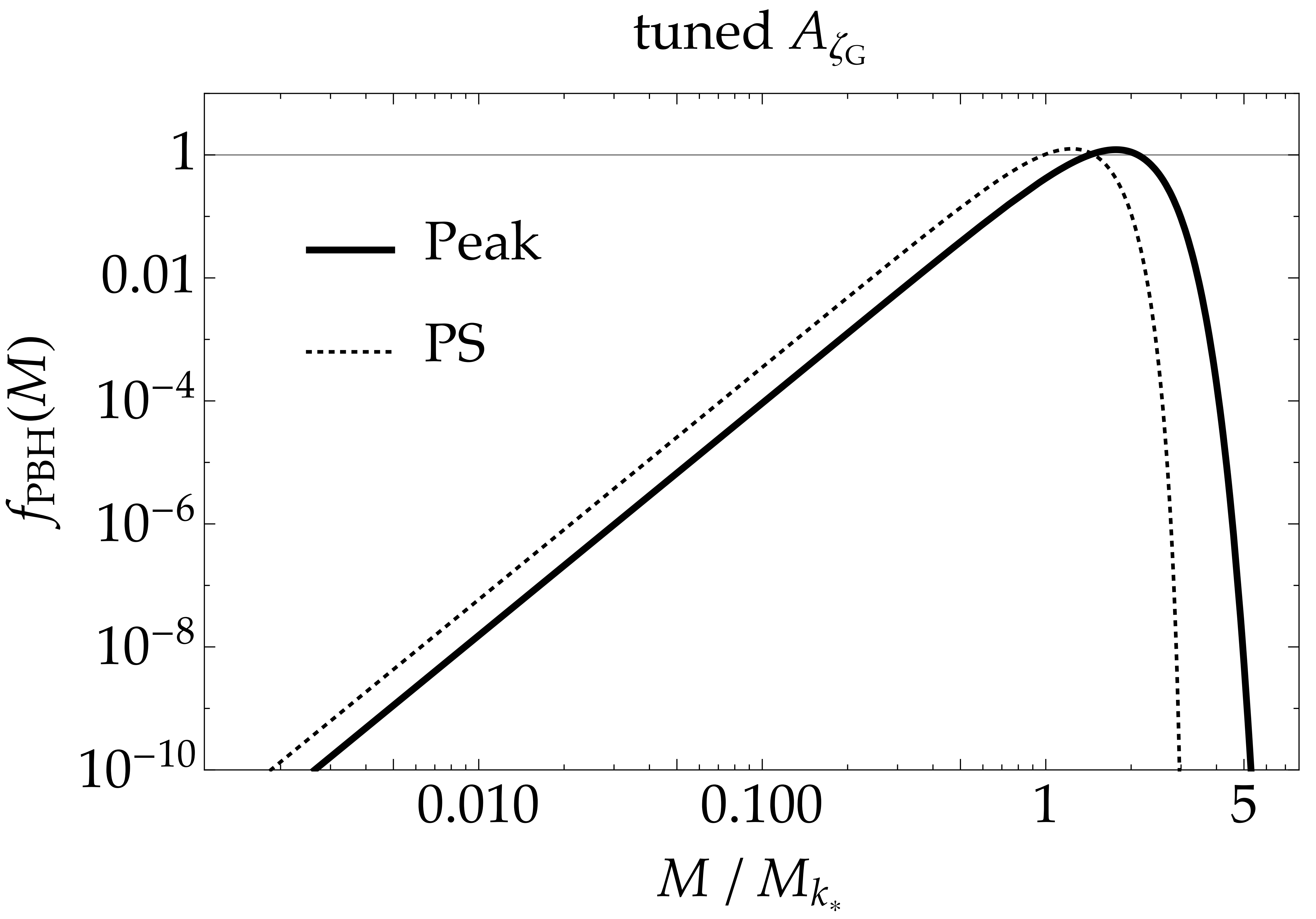
Musco, Miller, Polnarev '08

結果（ガウシアン）

Kitajima, YT, Yokoyama, Yoo '21

$$\mathscr{P}_{\zeta_G} = A_{\zeta_G}\delta(\log k - \log k_*)$$





$k_* = 1.56 \times 10^{13}\,\mathrm{Mpc}^{-1}$
$M_{k_*} = 10^{20}\,\mathrm{g}$

local-type NG

Yoo, Gong, Yokoyama '19

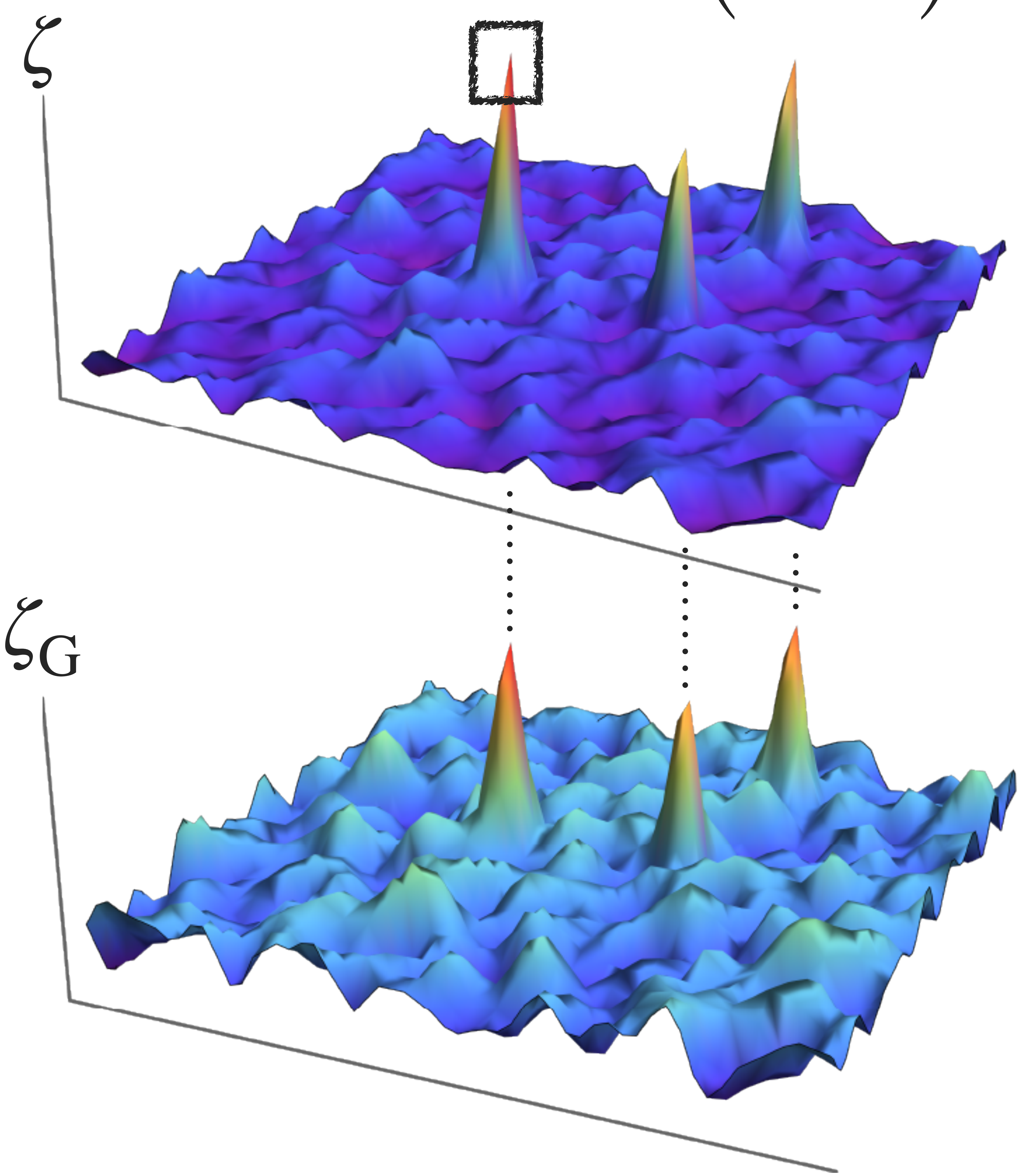
$$\zeta(\mathbf{x}) = \mathscr{F}_{\rm NG}\left(\zeta_{\rm G}(\mathbf{x})\right)$$

例えば・・・

- $\zeta = \zeta_{\rm G} + \frac{3}{5} f_{\rm NL}\zeta_{\rm G}^2$
- $\zeta = -\frac{1}{3}\log\left(1 - 3\zeta_{\rm G}\right)$: "exp-tail" in USR

$$= \zeta_{\rm G} + \frac{3}{5} \times \underbrace{\boxed{\frac{5}{2}}}_{f_{\rm NL}^{\rm USR}} \zeta_{\rm G}^2 + \cdots$$

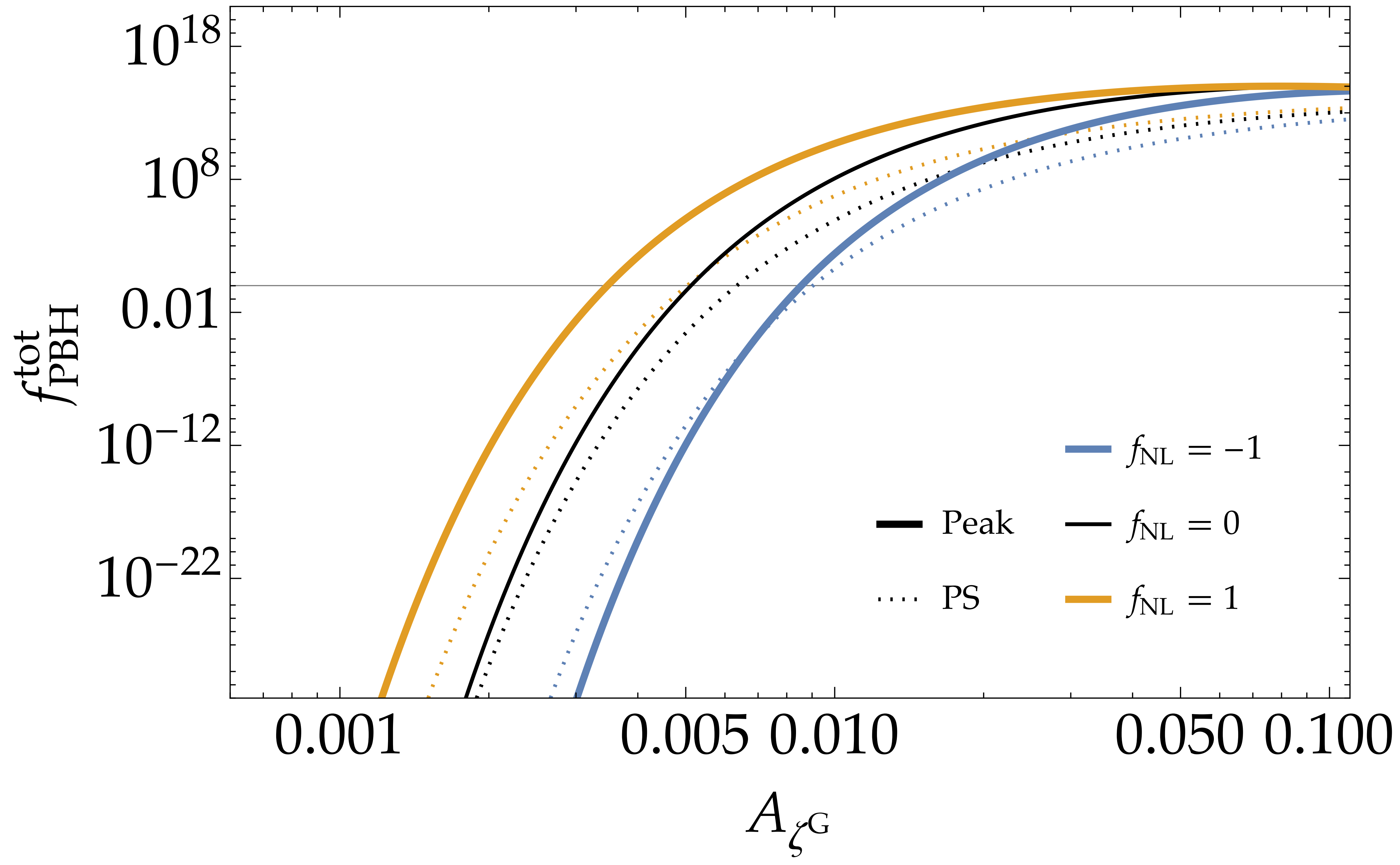
$$\hat{\zeta}(r) = \mathscr{F}_{\rm NG}\left(\hat{\zeta}_{\rm G}(r)\right)$$



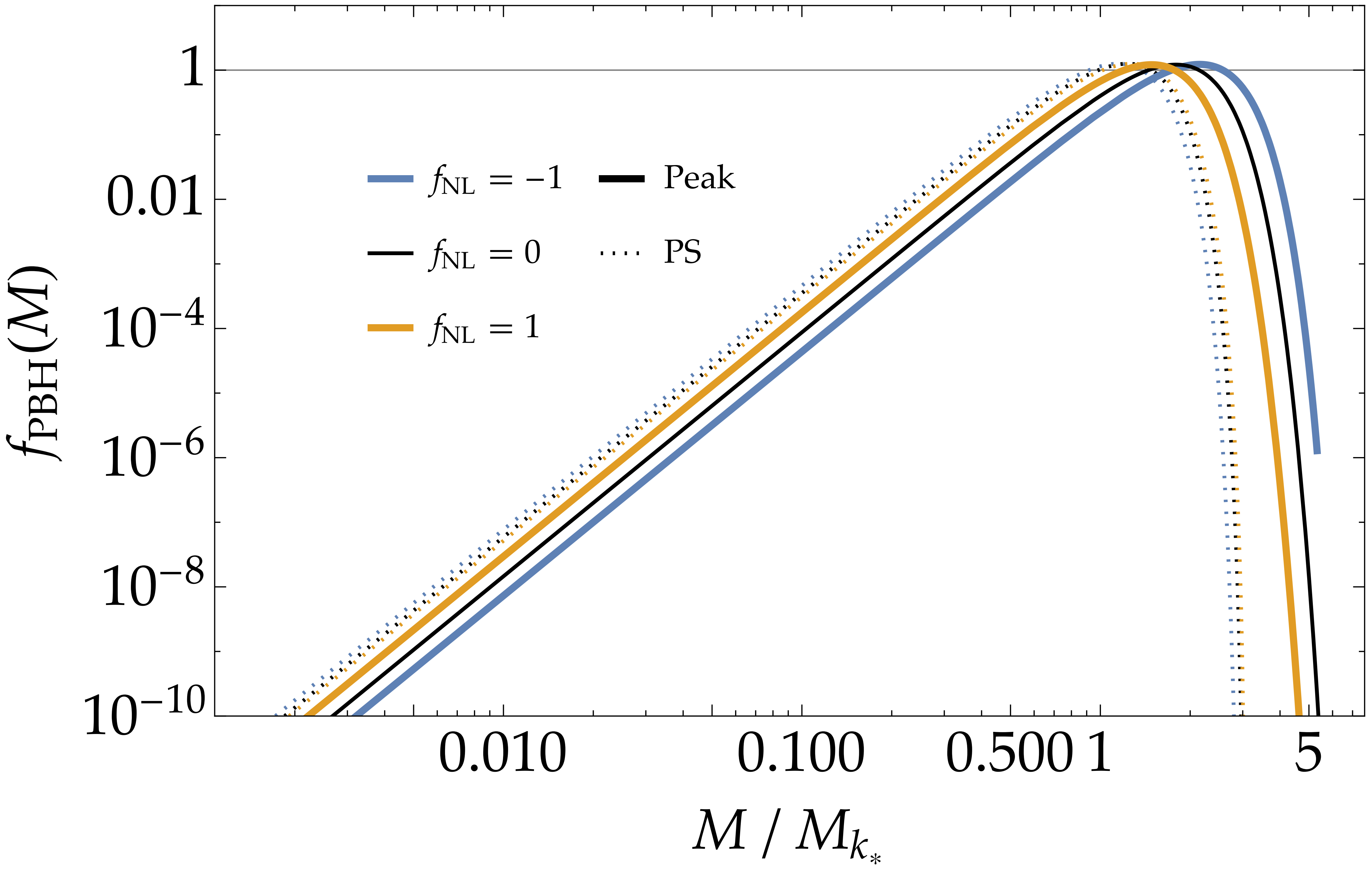
f_{NL}

Escrivà, YT, Yokoyama, Yoo '22

$$\mathscr{P}_{\zeta_G} = A_{\zeta_G}\delta(\log k - \log k_*)$$



tuned A_{ζ_G}



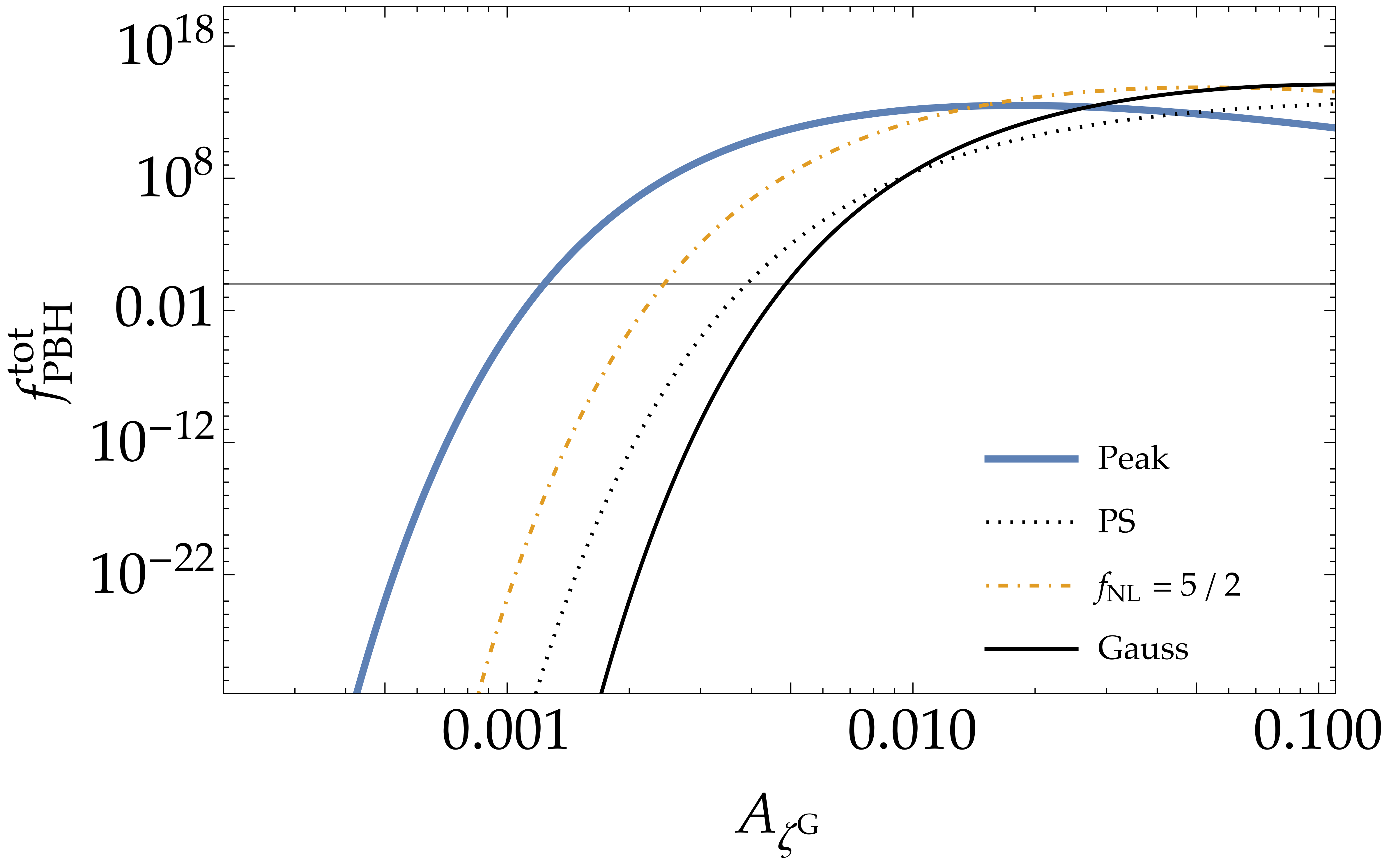
$k_* = 1.56 \times 10^{13}\,\mathrm{Mpc}^{-1}$

$M_{k_*} = 10^{20}\,\mathrm{g}$

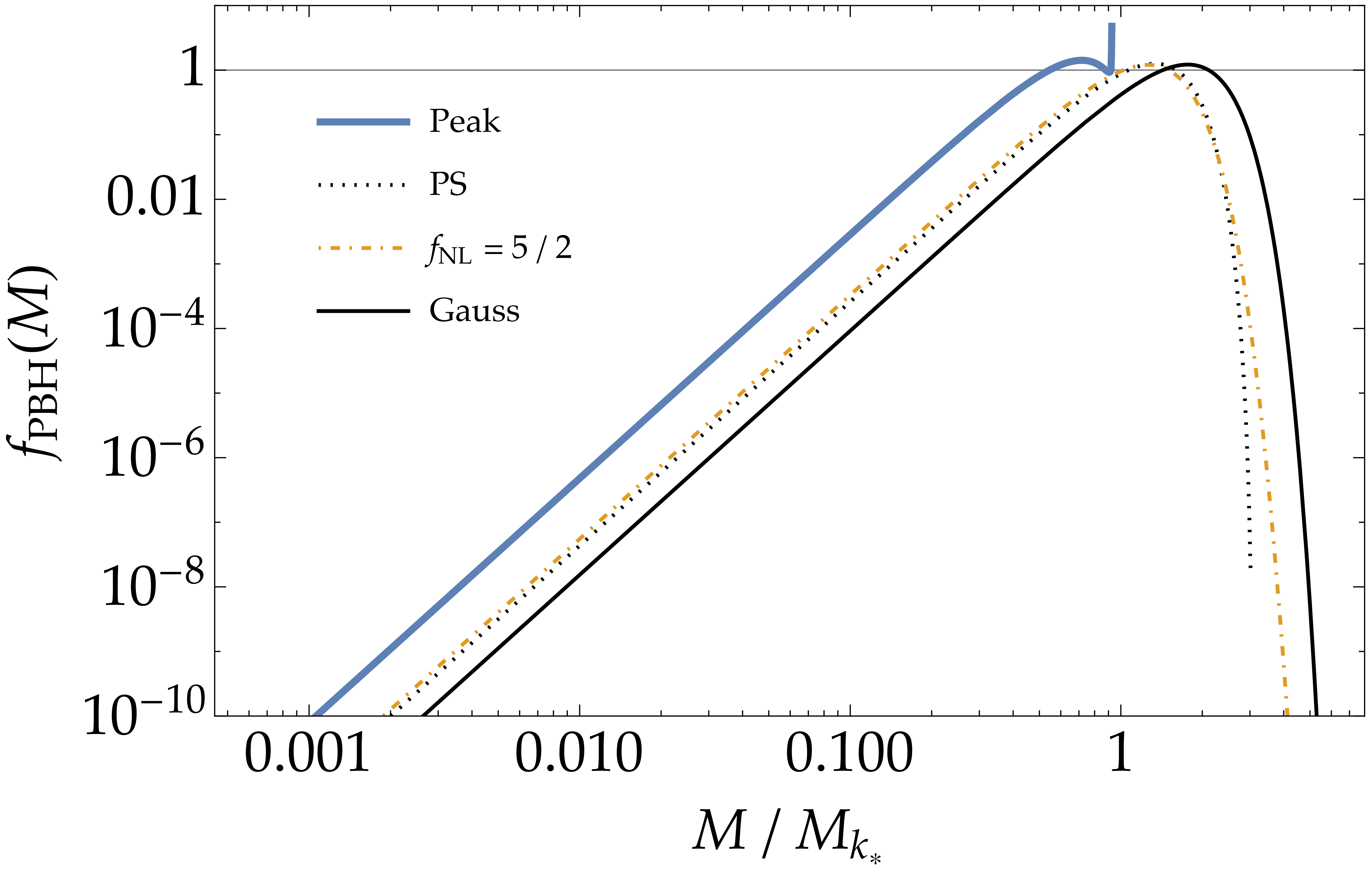
exp.-tail

Kitajima, YT, Yokoyama, Yoo '21

$$\zeta = -\frac{1}{3}\log\left(1 - 3\zeta_{\mathrm{G}}\right)$$



tuned $A_{\zeta_{\mathrm{G}}}$



$k_* = 1.56 \times 10^{13}\,\mathrm{Mpc}^{-1}$
$M_{k_*} = 10^{20}\,\mathrm{g}$

Induced GWs

誘導重力波

YT & Vennin '21

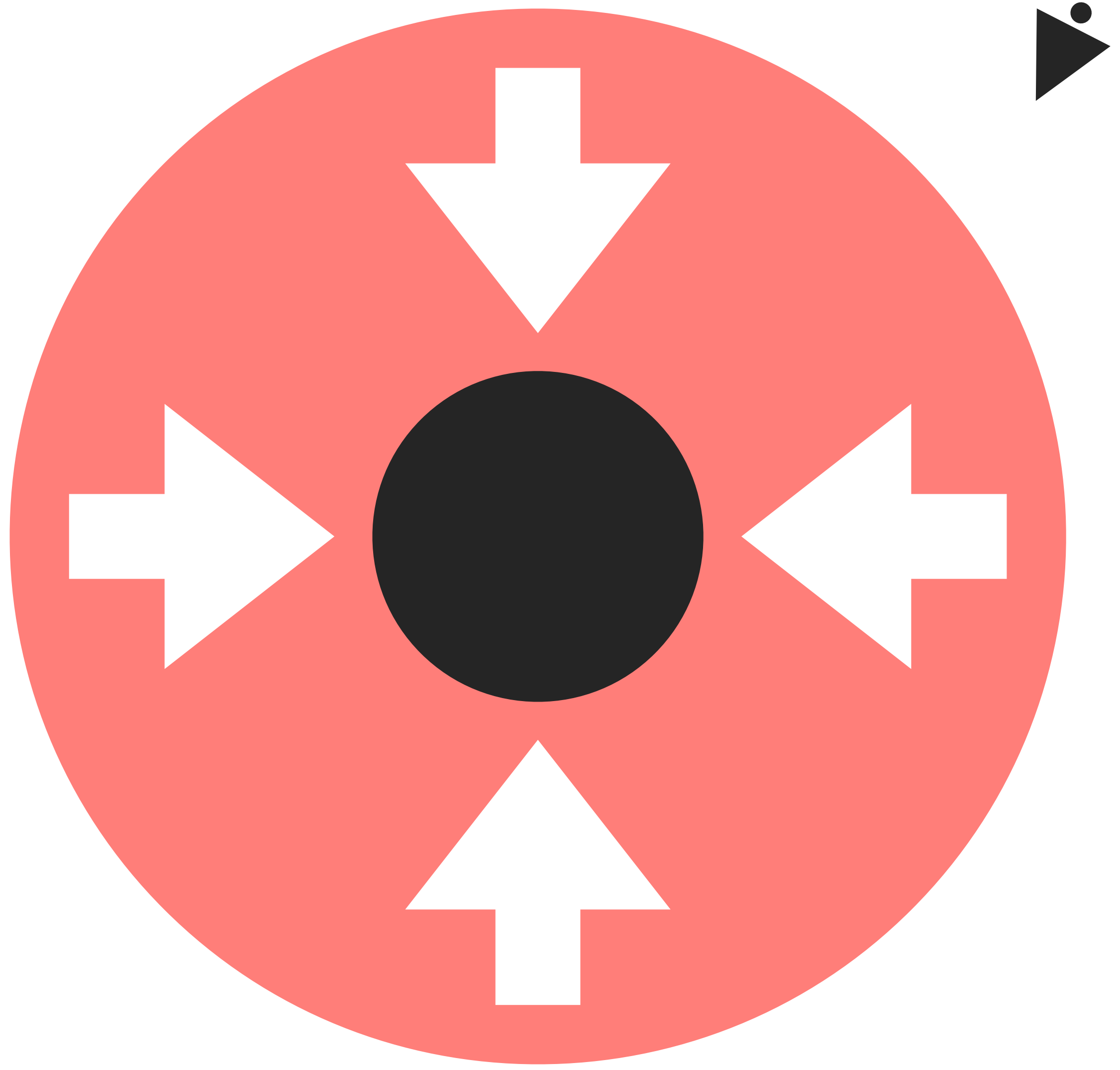
インフレーション

Escrivà, Kitajima, YT, Yokoyama, Yoo '21, '22

Abe, Inui, YT, Yokoyama in prep.

大きい初期ゆらぎ

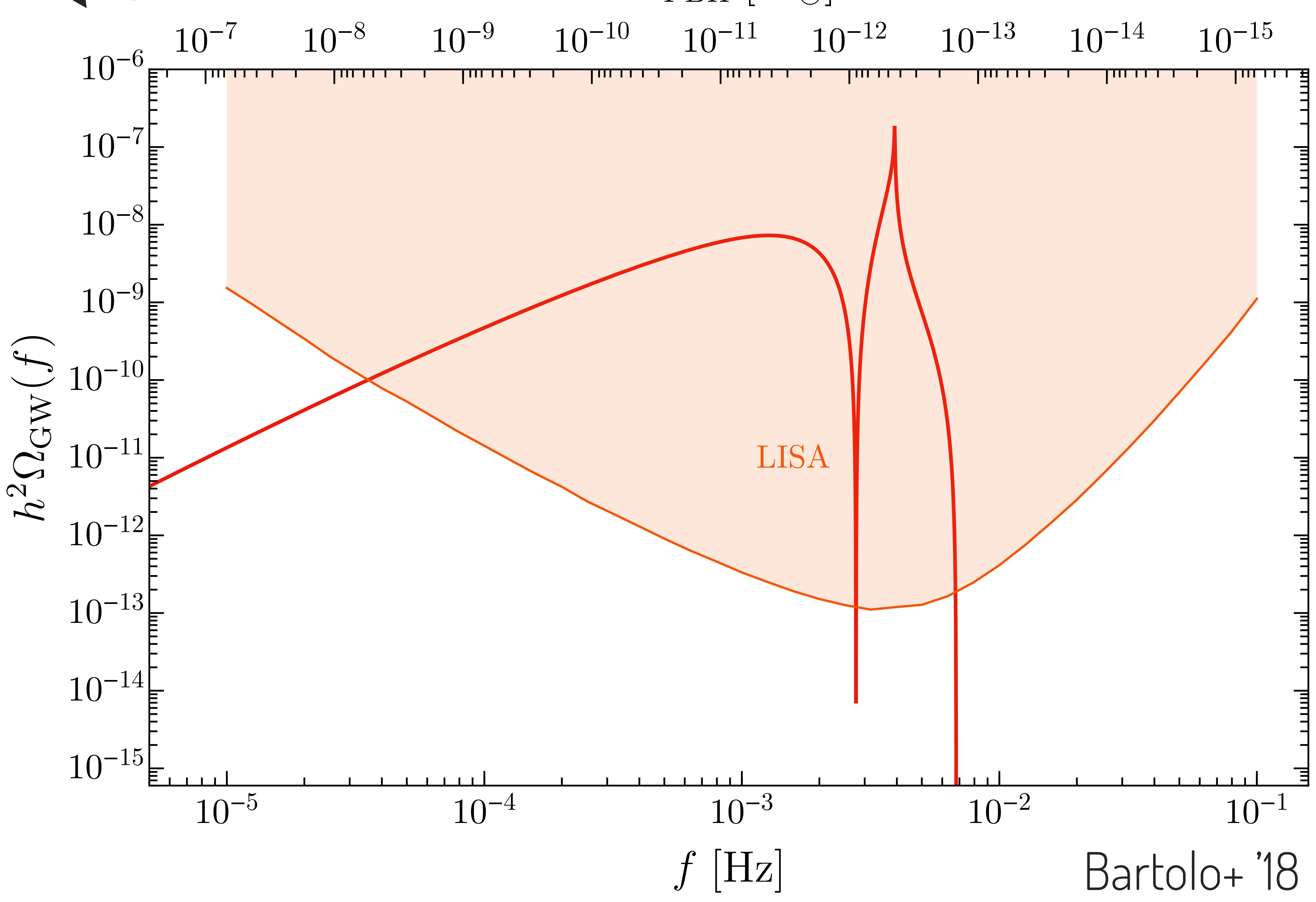
原始 BH



クロスチェック

2次誘導背景重力波

Saito & Yokoyama '08, '09



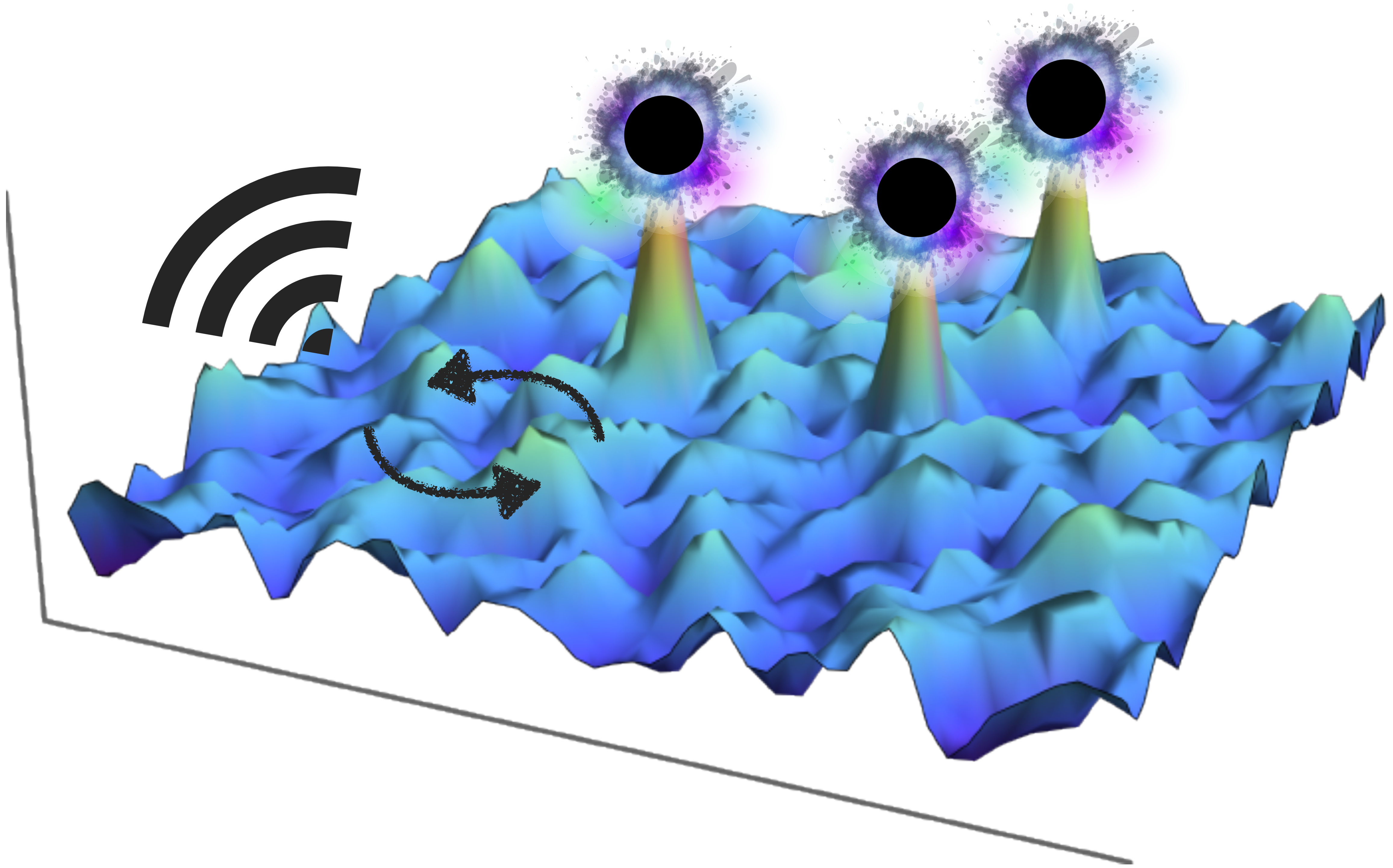
Bartolo+ '18

誘導重力波

YT & Vennin '21

インフレーション

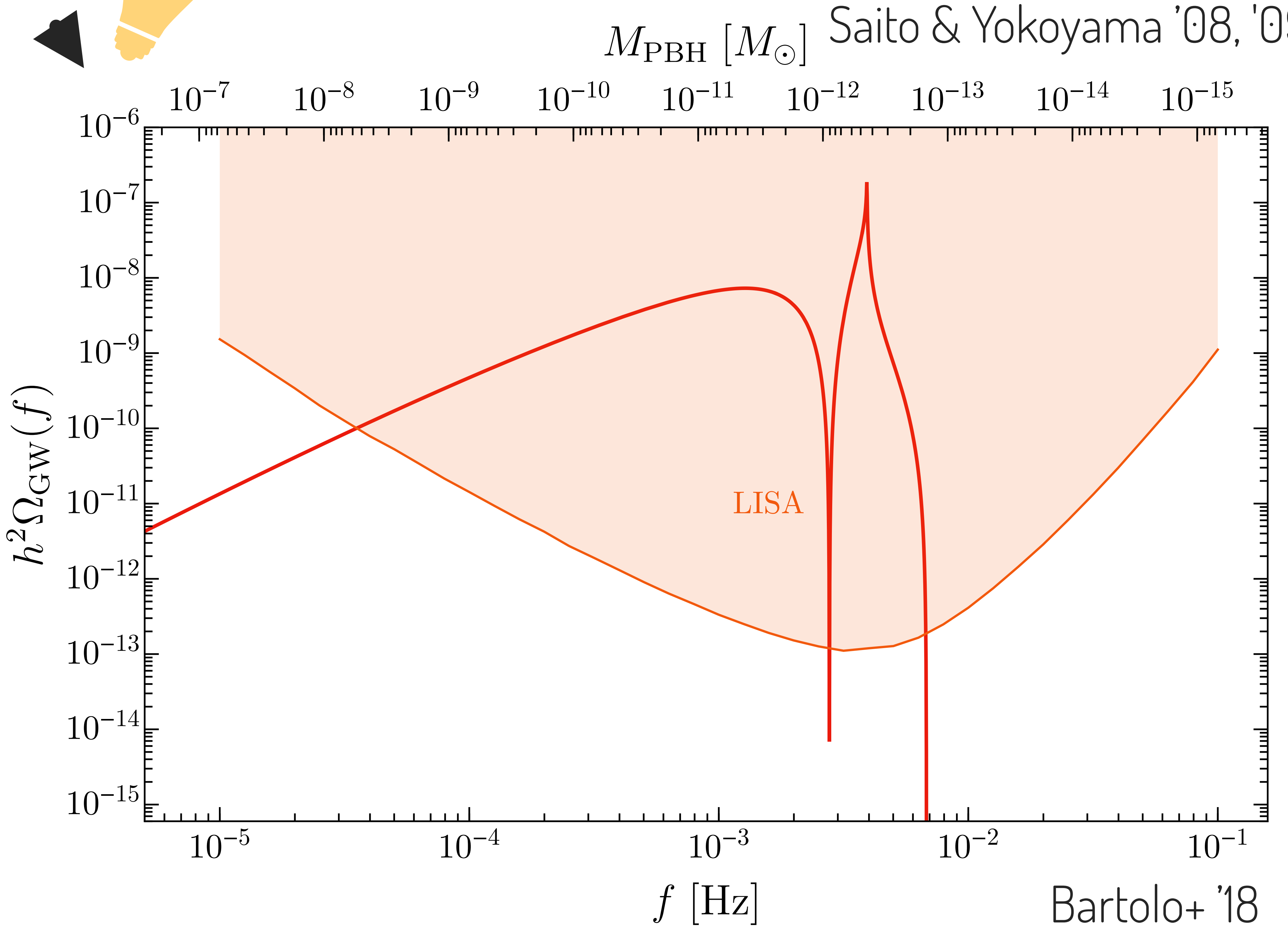
大きい初期ゆらぎ



Abe, Inui, YT, Yokoyama
in prep.

2次誘導背景重力波

Saito & Yokoyama '08, '09



Bartolo+ '18

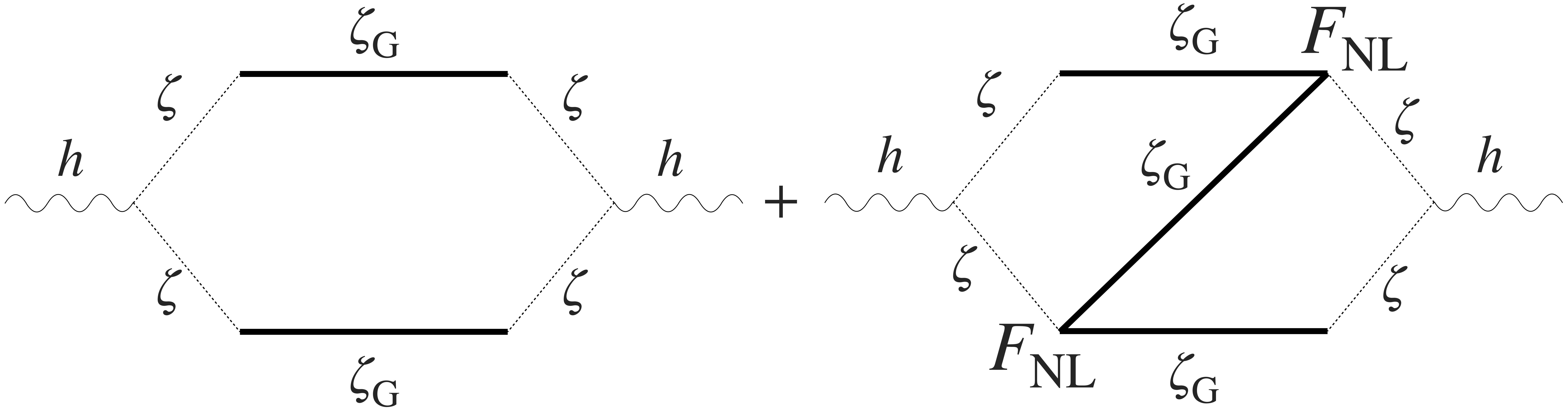
ダイアグラム

Adshead+ ‘21

$$\Box\, h_{\mathbf{k}} = S(\zeta_{\mathbf{p}}, \zeta_{\mathbf{k}-\mathbf{p}}) \;\Rightarrow h_{\mathbf{k}} = \int \mathrm{d}t'\, G_{\mathbf{k}}(t,t') S_{\mathbf{k}}(\zeta,\zeta;t')$$

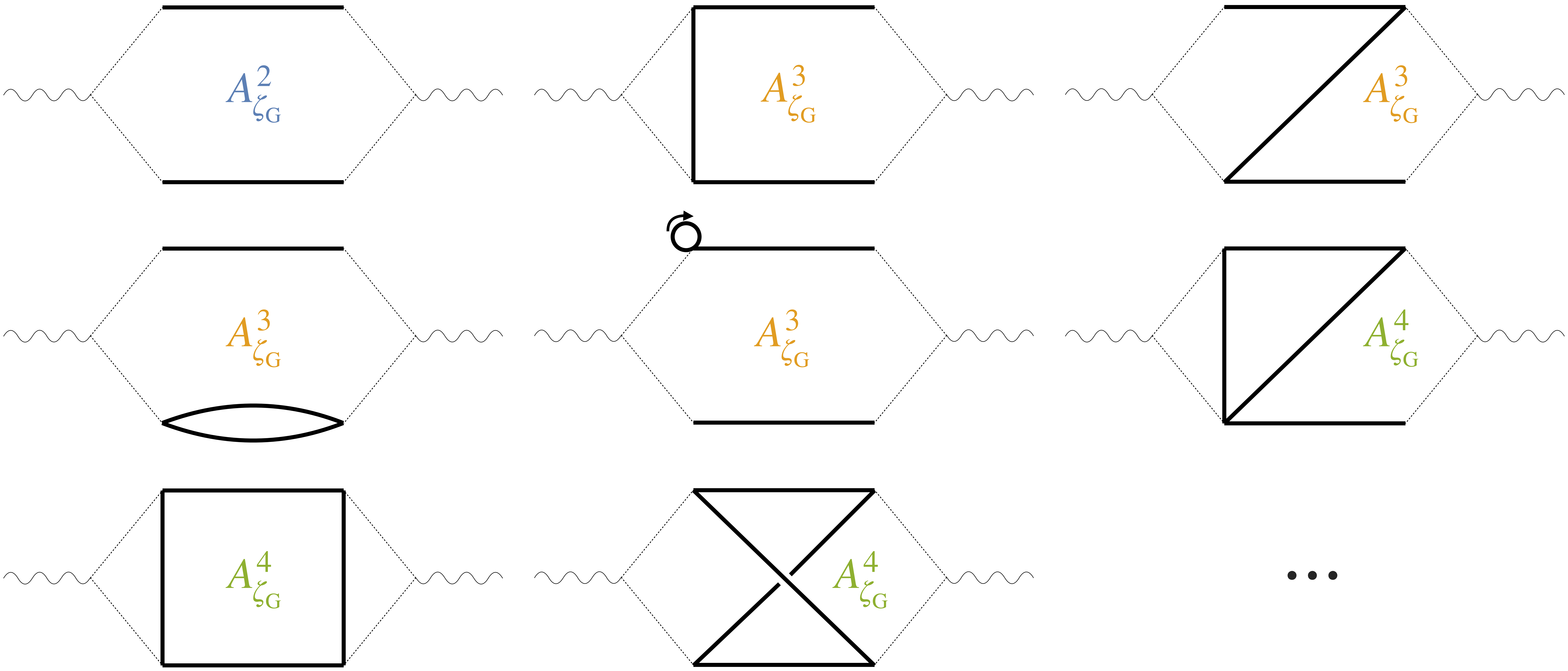
$$\Rightarrow \left\langle h_{\mathbf{k}} h'_{\mathbf{k}} \right\rangle = \int \mathrm{d}t'\mathrm{d}t'' G(t,t') G(t,t'') \left\langle S_{\mathbf{k}}(\zeta,\zeta;t') S'_{\mathbf{k}}(\zeta,\zeta;t') \right\rangle$$

$$\zeta = \zeta_{\mathrm{G}} + F_{\mathrm{NL}} \zeta_{\mathrm{G}}^2 + G_{\mathrm{NL}} \zeta_{\mathrm{G}}^3 + \cdots$$



ダイアグラム

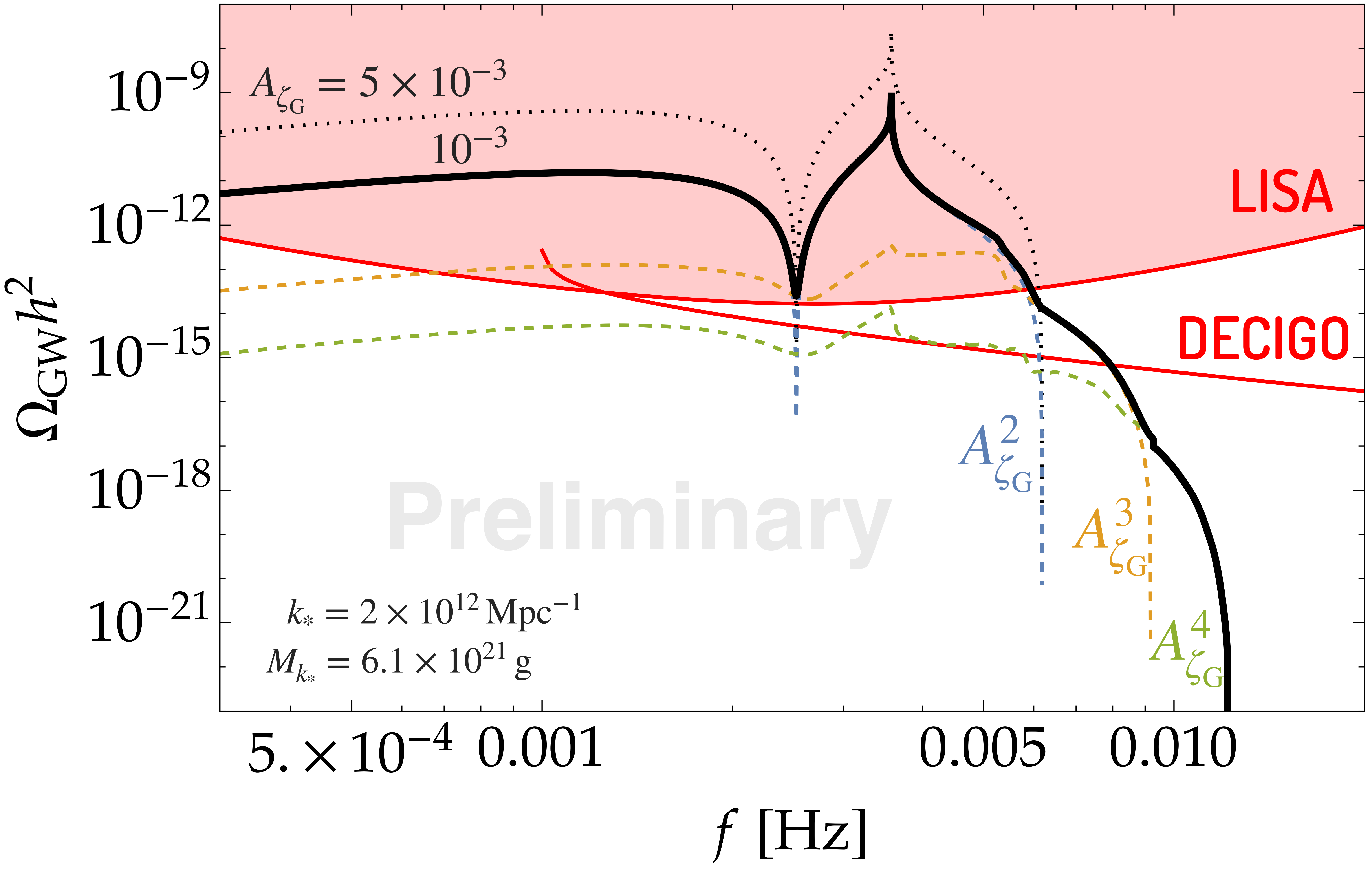
Abe, Inui, YT, Yokoyama in prep.



GW spectrum

Abe, Inui, YT, Yokoyama in prep.

$$\zeta = -\frac{1}{3}\log\left(1 - 3\zeta_{\mathrm{G}}\right) = \zeta_{\mathrm{G}} + \frac{3}{2}\zeta_{\mathrm{G}}^2 + 3\zeta_{\mathrm{G}}^3 + \cdots$$



$*\ h \sim \zeta^3, \zeta^4, \cdots$

cf. Yuan+ '19, Zhou '21

結論

- exp-tail : $\zeta \sim 1$ でガウシアンとは限らない
- 原始 BH のピーク理論
  - 質量のスケール則や ζ の非ガウス性にも対応
  - PS より非ガウス性の影響大
- 誘導重力波
  - exp-tail でも LISA で見える (heavier-tail だと…？ cf. Hooshangi+ '21, Cai+ '21)
  - UV 側に非ガウス性の情報？

補遺

$$\hat{g}(r) = \mu \left[\frac{1}{1-\gamma^2} \left(\psi(r) + \frac{1}{3} R_\bullet^2 \Delta\psi(r) \right) - k_\bullet^2 \frac{1}{\gamma(1-\gamma^2)} \frac{\sigma_0}{\sigma_2} \left(\gamma^2 \psi(r) + \frac{1}{3} R_\bullet^2 \Delta\psi(r) \right) \right]$$

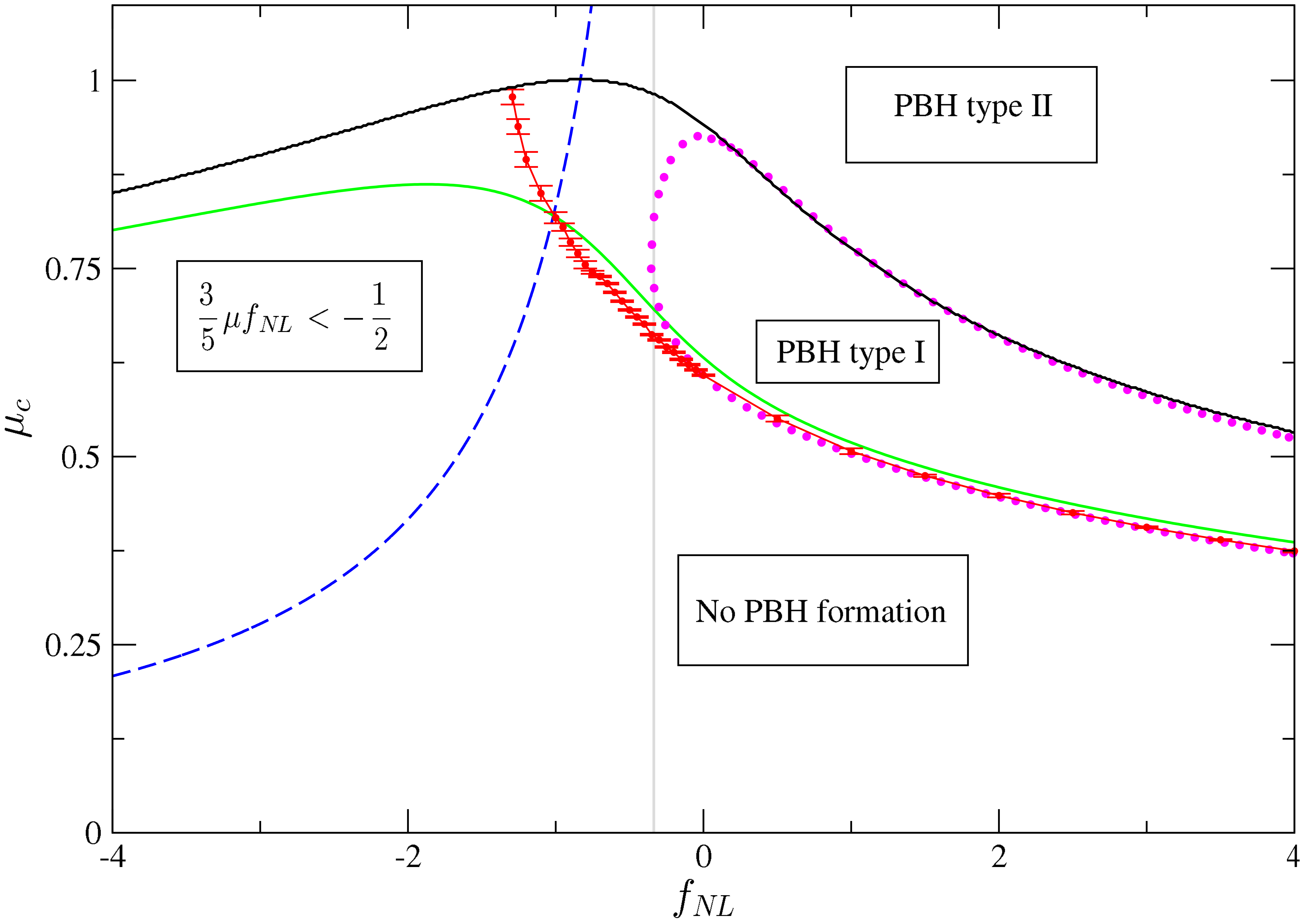
$$\sigma_n^2 = \int \frac{\mathrm{d}k}{k} k^{2n} \mathscr{P}_g(k), \quad \gamma = \frac{\langle k^2 \rangle}{\sqrt{\langle k^4 \rangle}}, \quad R_\bullet = \sqrt{\frac{3\langle k^2 \rangle}{\langle k^4 \rangle}} \qquad \psi(r) = \frac{1}{\sigma_0^2} \int \frac{\mathrm{d}k}{k} \frac{\sin kr}{kr} \mathscr{P}_g(k)$$

$$n_{\mathrm{pk}}(\mu, k_\bullet)\, \mathrm{d}\mu\, \mathrm{d}k_\bullet = \left[\frac{1}{V_\Omega} \int_\Omega \mathrm{d}^3 x \sum_{\nabla g(\mathbf{x}_\mathrm{p})=0} \delta^{(3)}(\mathbf{x} - \mathbf{x}_\mathrm{p}) \delta(\mu - \mu(\mathbf{x}_p)) \delta(k_\bullet - k_\bullet(\mathbf{x}_\mathrm{p})) \right] \mathrm{d}\mu\, \mathrm{d}k_\bullet$$

$$= \frac{2 \times 3^{3/2}}{(2\pi)^{3/2}} \mu k_\bullet \frac{\sigma_2^2}{\sigma_0 \sigma_1} f\left(\frac{\mu k_\bullet^2}{\sigma_2} \right) P_1 \left(\frac{\mu}{\sigma_0}, \frac{\mu k_\bullet^2}{\sigma_2} \right) \mathrm{d}\mu\, \mathrm{d}k_\bullet$$

$$f(\xi) = \frac{1}{2}\xi(\xi^2 - 3)\left(\mathrm{erf}\left[\frac{1}{2}\sqrt{\frac{5}{2}}\xi \right] + \mathrm{erf}\left[\sqrt{\frac{5}{2}}\xi \right] \right) + \sqrt{\frac{2}{5\pi}} \left\{ \left(\frac{8}{5} + \frac{31}{4}\xi^2 \right) \exp\left[-\frac{5}{8}\xi^2 \right] + \left(-\frac{8}{5} + \frac{1}{2}\xi^2 \right) \exp\left[-\frac{5}{2}\xi^2 \right] \right\} \qquad P_1(\nu, \xi) = \frac{1}{2\pi\sqrt{1-\gamma^2}} \exp\left[-\frac{1}{2} \left(\nu^2 + \frac{(\xi - \gamma\nu)^2}{1-\gamma^2} \right) \right]$$

補遺



PBH type II
PBH type I
No PBH formation
$\frac{3}{5}\mu f_{NL} < -\frac{1}{2}$
μ_c
f_{NL}
1
0.75
0.5
0.25
0
-4
-2
0
2
4

補遺

$$\zeta = -\frac{1}{3}\log\left(1 - 3\zeta_{\mathrm{G}}\right)$$